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Calculation of Dynamic Impedance of Foundations Using Finite Element Procedures

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Calculation of Dynamic Impedance of Foundations Using Finite Element Procedures

by C. Coronado and N. Gidwani

Synopsis: The analysis and design of foundations under dynamic loads due machinery is routinely conducted in industrial projects. Determination of the dynamic impedance of the foundation is required for performing such analysis and design. Different tools are available for the calculation of foundation impedances including: published closed form solutions, charts, tables and specialized computer codes. Despite the convenience of these tools, their use for production work is cumbersome. The main reason being that, in its simplest form, a dynamic impedance is provided as a frequency dependent complex function or as a spring-dashpot system, with the spring becoming negative for certain frequencies; which cannot be directly implemented in standard structural analysis codes. The use of such impedances requires a clear understanding of the theory behind their calculation along with several principles of soil dynamic, which are not covered in the regular curriculum of structural engineering programs. This paper aims to fill this gap by providing structural engineers with the basic tools for the understanding, calculation and use of foundation impedance functions. For this purpose, numerical examples are provided to illustrate the application of the approaches discussed in this paper for the calculation and application of dynamic impedances.

Keywords: dynamic impedance; dynamic loads; finite element; foundation; machine; soil

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1. INTRODUCTION

The analysis and design of foundations under dynamic loads due to rotating machinery is routinely conducted in industrial projects. Determination of the dynamic behavior of the soil-foundation system is required for performing such analysis and design. A key aspect of this process involves modeling the soil behavior under excitation introduced by the rotating equipment. In current industry practice, the soil response is modeled using the dynamic stiffness/impedance concept. In general, for rigid surface foundations the dynamic impedance is given by six frequency dependent springs and dampers. For flexible or embedded foundations the dynamic impedance is given by a complex frequency dependent matrix.

Different tools are available for the calculation of the dynamic impedances including closed form solutions, charts, tables and specialized computer codes. Despite the usefulness of these tools, their application for production work is sometimes cumbersome when using standard structural analysis codes since the dynamic impedance is provided as a complex number, complex matrix or in form of a spring and damper, with the spring becoming negative for certain frequencies. The use of such impedances requires a clear understanding of the theory behind their calculation and the principles behind modeling of the dynamic response of soil, both of which are not included in the regular curriculum of structural engineering programs. Therefore, the purpose of this paper is to provide structural engineers with basic tools for the understanding, calculation and use of impedance functions using standard finite element codes.

In particular, the definition of the dynamic impedance is presented and its application is illustrated using simple foundation models. Several procedures are discussed to model the unbounded soil media using finite elements (FE). It must be noted that in a FE model only a fraction of the soil can be discretized; therefore, appropriate boundary conditions (non-reflective boundaries) must be applied where the soil is arbitrarily truncated. Recommendations are provided to select the element size, element type, to obtain the dynamic impedance of surface foundations. Finally, numerical examples are included to illustrate the application of the approaches discussed in this paper for the calculation of dynamic impedances of surface foundations.

2. DYNAMIC IMPEDANCE DEFINITION

The response of a rigid foundation to static or dynamic load arises solely from the deformation of the supporting soil. The static soil stiffness ($K = P/U$) is used to model the soil-foundation response to static load. In an analogous manner, the dynamic soil impedance/stiffness ($K = P(t)/U(t)$) is used to model the soil-foundation response to dynamic loads (e.g. due to machinery operation). In particular, six dynamic impedances are required, three translational and three rotational, to formulate the dynamic equilibrium equation of a rigid foundation, as shown in Figure 1. These impedances are a function of the foundation geometry, the soil properties and vibration frequency of the machine (f_m, ω_m).

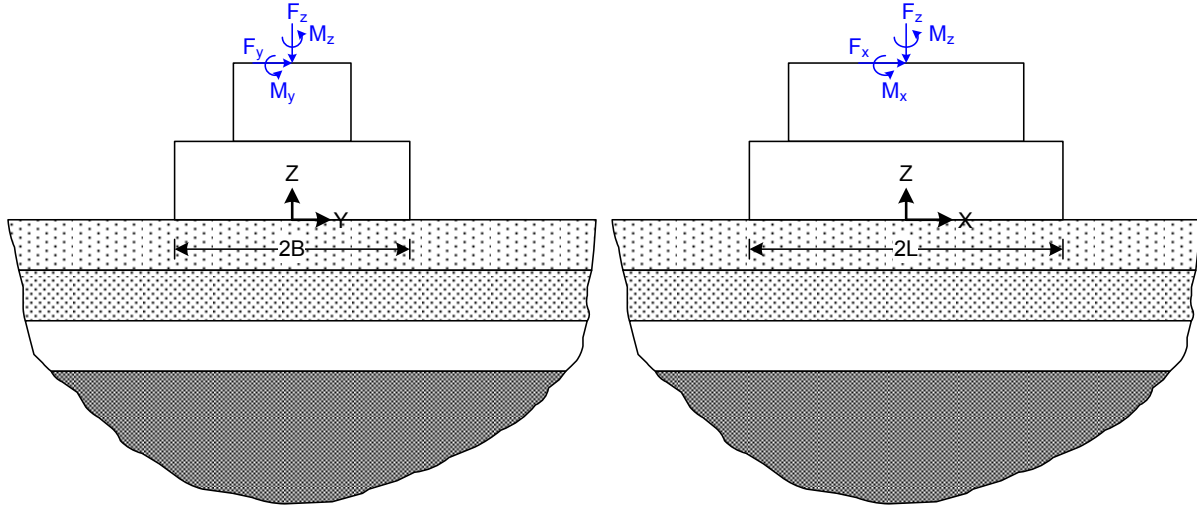


Figure 1—Degrees of freedom for rigid foundation block resting on a layered half-space.

The procedure used to calculate the dynamic impedances of a rigid surface foundation can be summarized in the following steps:

1. The foundation is modeled as massless and infinitely rigid; therefore, only the geometry of the area in contact with the soil is required (e.g., B and L shown in Figure 1). The use of a massless foundation is important since it avoids the need for recalculating the dynamic impedance every time that the foundation mass changes, which often happens during the design process.
2. A harmonic force or moment of frequency ω and of unit magnitude is applied to the rigid foundation (e.g., $P(t) = P_0 e^{i\omega t}$ or $M(t) = M_0 e^{i\omega t}$), as shown in Figure 2. Such force/moment generates stress waves that propagate into the underlying soil, which is modeled as a viscoelastic material. Therefore, the following properties are required for each soil layer: thickness (h), Modulus of elasticity (E_s), Poisson's ratio (ν), density (ρ), and material damping (ζ).
3. The steady state vibration amplitude ($U(t) = U_0 e^{i\omega t + i\phi}$ or $\theta(t) = \theta_0 e^{i\omega t + i\phi}$) of the foundation under the harmonic force is obtained by keeping track of the reflections and refractions that take place every time that the stress waves reach a soil layer boundary. This is achieved by finding the different wave paths shown in Figure 2.
4. The dynamic impedance $K(\omega)$ is defined as the ratio between the harmonic force acting on the foundation and its vibration amplitude as shown in Eq. (1). It must be noted that this is a frequency dependent complex quantity.

$$K(\omega) = \frac{P(t)}{U(t)} = \frac{P_0 e^{i\omega t}}{U_0 e^{i\omega t + i\phi}} = \frac{P_0}{U_0} e^{-i\phi} \quad (1)$$

5. In soil dynamics, it is customary to express the complex dynamic impedance as shown in Eq. (2). In addition, the real and imaginary parts of the dynamic impedance are associated, by analogy, with a dynamic (frequency dependent) *spring* and *dashpot* as shown in Eq. (3).

$$K(\omega) = \bar{k} + i\omega C \quad (2)$$

$$\bar{k}(\omega) = \text{Re}(K(\omega)) = \frac{P_0}{U_0} \cos \phi \quad (3)$$

$$C(\omega) = \frac{\text{Im}(K(\omega))}{\omega} = -\frac{P_0}{\omega U_0} \sin \phi$$

6. Steps 2 to 5 are repeated for each frequency ω_i of interest, until the range of vibration frequencies of the machine is covered.

The above approach can also be used for calculating the dynamic impedance of a single pile or pile foundation. However, in this case, the mass and flexibility of each pile are considered. Similarly, the concept of dynamic impedance can be extended to flexible foundations; however, in this case, a dynamic impedance matrix is required. Details regarding the generalization of the dynamic impedance concept can be found elsewhere, e.g. Refs. [5], [7] and [8].

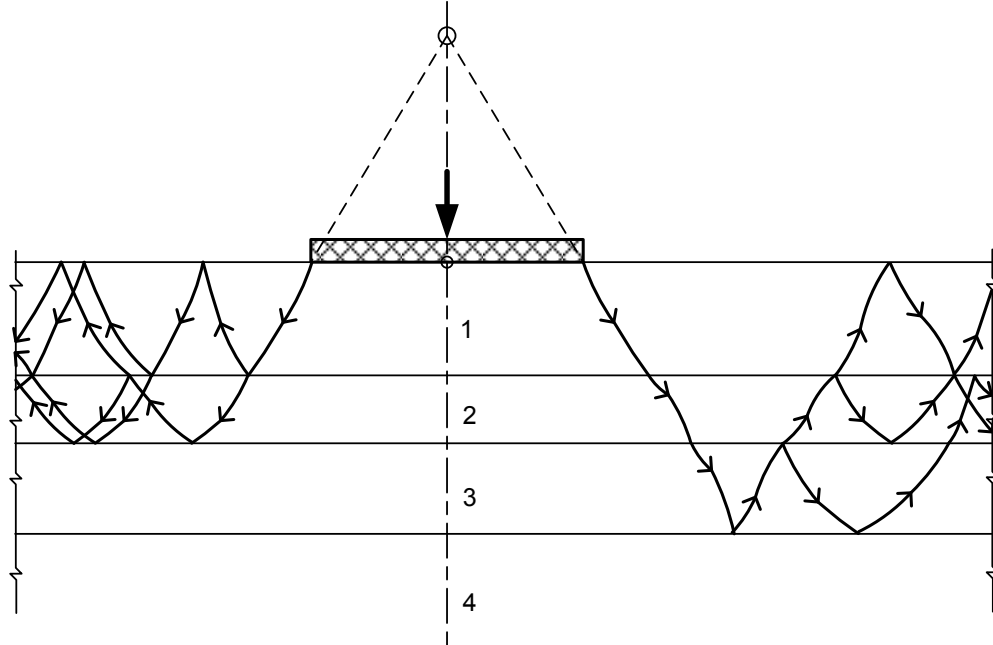


Figure 2—Downwards and upwards wave propagation for surface disk in layered half-space, according to Ref [6].

3. CALCULATION OF THE SOIL DYNAMIC IMPEDANCE/STIFFNESS

There are several approaches for calculating the frequency dependent dynamic impedance of a soil supported foundation; these include: (a) Published closed form solutions, tables and charts (Refs. [1], [2] and [3]); (b) Simplified methods also known as strength of material approaches (Refs. [5] and [6]); and (c) Computer-based numerical analysis methods such as the boundary element method, finite element method, Green's functions and integral equations among others (Ref. [7], [8], [9] and [12]).

A detailed discussion of above procedures is beyond the scope of this paper. However, it must be noted that closed-form solutions are limited in scope since they just apply to foundations resting on homogeneous or simplified soil

profiles. Given that typical machine foundations are supported on layered soil profiles, computer based analyses are required for the calculation of the dynamic impedance of the subject foundations. Therefore, the rest of this section is devoted to discussing the procedures used for the calculation of foundation impedances using the finite element method. The purpose is to provide the background required for understanding the calculation of foundation impedances. However, it is expected that impedance calculations for production work will be conducted using computer codes specially designed for that purpose, e.g., Ref. [13].

Finite Element Calculation of Dynamic Foundation Impedances

For complex foundation geometries or soil conditions, the dynamic soil impedance can be determined by dynamic analysis of a three-dimensional or two-dimensional continuum model of the soil-foundation system. In particular, the six steps detailed in Section 2 can be implemented using the finite element (FE) method. In this case, the soil is modeled as an elastic or viscoelastic material, which can be considered isotropic, anisotropic, homogeneous or nonhomogeneous.

The general approach presented in this section consists of modeling the soil with axisymmetric or 3D solid elements. When possible, axisymmetric elements should be used since they greatly reduce the modeling and analysis time compared to that of an equivalent model using 3D-solid elements.

Non-reflective Boundaries—In a FE model, only a portion of the soil (i.e., a soil island) can be discretized; therefore, appropriate boundary conditions (non-reflective boundaries) must be applied where the soil is arbitrarily truncated. While effective, quiet, non-reflecting or transmitting boundaries have been developed in the literature for this purpose, they have not yet been implemented in most computer codes used for structural analysis. This section discusses some of the boundaries that can be easily implemented in such codes. Such boundaries are approximate in nature; nevertheless, FE models based on them converge to the theoretical elastodynamics solution as the soil island size is increased. In general, results within 5% of theoretical solutions can be achieved with very reasonable (i.e., small) soil island sizes. Specific details regarding the many different boundary types proposed in the literature can be found in Refs. [5], [9] and [10].

Gradual Damping Boundary Elements—this method uses conventional finite elements to model an unbounded domain. In particular, the infinite domain is approximated by attaching artificially damped elements outside the area of interest in the analysis. The structural damping of the attached elements is gradually increased in order to model the dispersion of the propagating waves in the unbounded media. For instance, in the axisymmetric model shown in Figure 3, the unbounded media is arbitrarily truncated and divided in internal, transition and gradually damped regions.

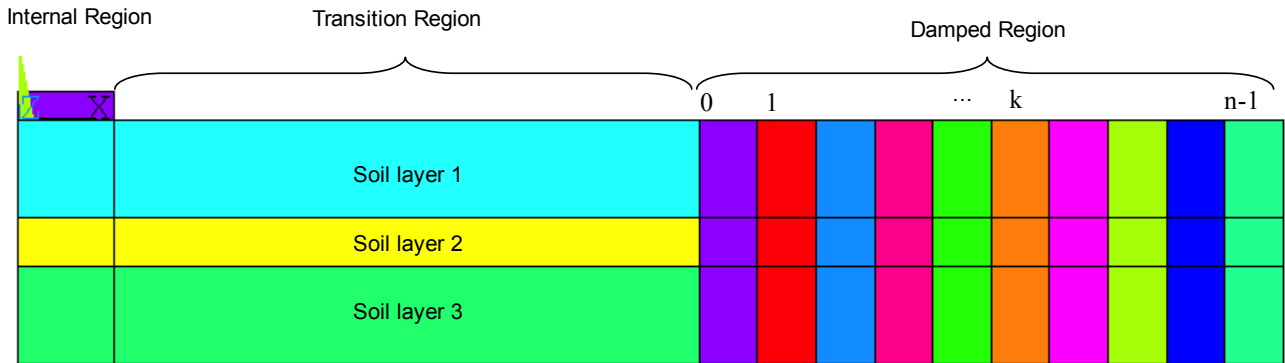


Figure 3—Use of gradual damping to model a layered unbounded media.

The damping of the elements in the damped region is gradually increased from the innermost set to the set next to the finite boundary. In particular, the gradual increase proposed in Ref [12] and given by Eq. (4) is recommended here; where, α_0 is the damping ratio for the first damped element, ζ is a constant factor larger than one which controls the gradual damping growth, and $k = 0, 1, 2, \dots, n-1$ with 0 corresponding to the innermost set of damped elements, and $n-1$ to the last, as shown in Figure 3. In a computer code capable of conducting frequency

domain analyses, this exponential increase of damping can be easily included using the complex modulus of elasticity given by Eq. (5).

$$\xi = \alpha_0 \zeta^k \quad (4)$$

$$E_k = E(1 + i2\alpha_0 \zeta^k) \quad (5)$$

The exponential function defined by Eq. (4) ensures a gradual rate of increase in damping. This prevents a sudden damping increase that by itself could cause reflection of the propagating waves. The following criteria should be followed (Ref [12]) for achieving an optimum damping increase:

- Sufficient damping such that the effect of the boundary is negligible.
- Damping is gradual enough such that there is no reflection caused by a sudden damped condition.

Spring-dashpot boundaries—Simple boundaries approximating the half-space can be specified in the form of frequency independent springs and dashpots (Ref. [9]). The springs are meant to model the elastic response of the surrounding media while the dashpots are intended to account for the radiation of waves propagating towards infinite. Due to the local nature of these boundaries, their energy absorption capabilities depend not only on material properties but also on the frequency content of the excitation. These spring-dashpot elements are attached to the boundary nodes of the soil island. The procedures used to define the spring-dashpot properties for rectangular soil islands are described next; additional details are provided in Ref. [9].

The dashpot coefficients are determined in terms of the material properties of each soil layer per Ref [11]. In particular, the normal ($C_n = \rho V_p$) and tangential ($C_t = \rho V_s$) dashpot coefficients are a function of the density (ρ), the compressional wave velocity (V_p) and the shear wave velocity (V_s) of the soil layer. The springs are determined in terms of the material properties and geometry of the soil island (Ref. [9]). For a rectangular soil island, the spring constants are calculated per Eqs. (6) to (8); where, $R = \sqrt{x^2 + y^2 + z^2}$, G is the shear modulus, and ν is Poisson's ratio.

$$k_{xx} = \frac{2Gx}{R^2} \left(\frac{3x^2 + (1-2\nu)R^2}{x^2 + (3-4\nu)R^2} \right), k_{xy} = \frac{2Gx}{R^2} \left(\frac{3y^2 + (1-2\nu)R^2}{y^2 + (3-4\nu)R^2} \right), k_{xz} = \frac{2Gx}{R^2} \left(\frac{3z^2 + (1-2\nu)R^2}{z^2 + (3-4\nu)R^2} \right) \quad (6)$$

$$k_{yx} = \frac{2Gy}{R^2} \left(\frac{3x^2 + (1-2\nu)R^2}{x^2 + (3-4\nu)R^2} \right), k_{yy} = \frac{2Gy}{R^2} \left(\frac{3y^2 + (1-2\nu)R^2}{y^2 + (3-4\nu)R^2} \right), k_{yz} = \frac{2Gy}{R^2} \left(\frac{3z^2 + (1-2\nu)R^2}{z^2 + (3-4\nu)R^2} \right) \quad (7)$$

$$k_{zx} = \frac{2Gz}{R^2} \left(\frac{3x^2 + (1-2\nu)R^2}{x^2 + (3-4\nu)R^2} \right), k_{zy} = \frac{2Gz}{R^2} \left(\frac{3y^2 + (1-2\nu)R^2}{y^2 + (3-4\nu)R^2} \right), k_{zz} = \frac{2Gz}{R^2} \left(\frac{3z^2 + (1-2\nu)R^2}{z^2 + (3-4\nu)R^2} \right) \quad (8)$$

The simple boundaries described in this section can be equally applied in frequency and time domain analyses (linear and nonlinear). Furthermore, they can be combined with the gradual damping elements described in the previous section.

Modeling Considerations—the running time and accuracy of a finite element solution are greatly affected by the mesh quality, in other words by the element type, size, shape, and aspect ratio used during the modeling stage. General FE computer codes (e.g., Ref. [4]) offer extensive element libraries and a full array of modeling techniques that help to produce state of the art finite element models. For a detailed description of such techniques the reader is referred to the Theory, Modeling and User's manual of the FE code selected for the application. The present discussion is limited to the use of linear elastic axisymmetric and 3D solid finite elements to model the soil in the frequency domain, as illustrated in Section 4.

Element Size—several recommendations are provided in the literature in order to set the element size for wave propagation analyses using finite elements. The general concept is that the mesh should be fine enough to

resolve the propagating wave. Overall, recommendations provided in the literature range between 5 and 20 elements per wavelength ($\lambda = 2\pi V_s/\omega$). Of course, better results should be expected from highly refined models; nevertheless, they are computationally expensive and may be impractical from an engineering point of view. Therefore, a compromise must be reached among discretization quality and solution efficiency. In this study, it is recommended to start with a coarse mesh, about five elements per wavelength ($E_{\text{size}} \sim \lambda/5$) along the direction of the wave and investigate the effect of element size on the solution quality using an axisymmetric model of the foundation under study.

Soil Island Size—the size of the portion of soil modeled affects the accuracy achieved when calculating the dynamic impedance of a foundation. In Figure 4, a typical soil island is presented for illustration purposes. Gradual damping is used to model the attenuation of stress waves as discussed in the section dealing with non-reflective boundaries.

In order to define the size of the soil island, it is recommended to use a transition region (L_T) at least equal to the foundation radius ($L_T > r_f$) and a transition region (L_H) at least equal to four times foundation radius ($L_H > 4r_f$). Alternatively, the total depth (H) can be adjusted as a function of the maximum Rayleigh wave length ($\lambda_R \sim 2\pi V_s/\omega_{\min}$); it is recommended to use $H > 2\lambda_R$ since Rayleigh waves almost vanish at this depth.

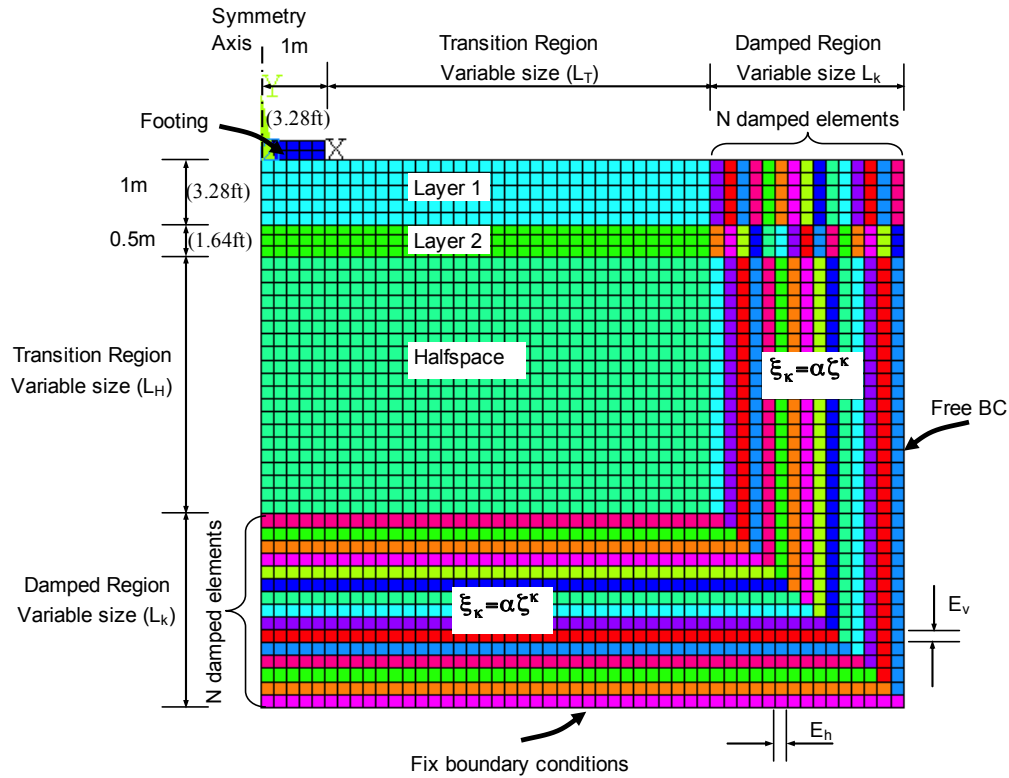


Figure 4—Axisymmetric finite element model of a surface foundation.

4. APPLICATION EXAMPLES

The purpose of this section is to illustrate the steps required for calculating the dynamic impedance of surface foundations, using the finite element method, along with their application for calculating the dynamic response of a typical machine foundation.

Dynamic Impedance of Rigid Disk on Layered Soil Profile

Figure 5 shows the soil properties and foundation geometry used in this example. The soil profile has a rigid soil layer near the surface (fixed base), which results in total reflection of the vertical waves produced by the vibration of the foundation. Such reflections along with the layered soil profile result in a complex dynamic response, which requires solution using numerical methods as discussed here.

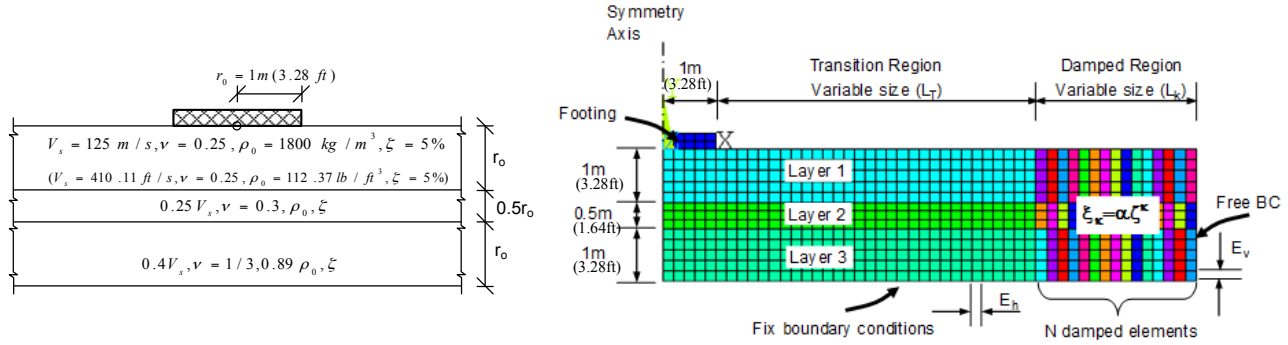


Figure 5—Soil profile and axisymmetric finite element model of the surface foundation.

The finite element model of the foundation is shown in Figure 5. Axisymmetric elements are used to model the soil and foundation. The radiation of stress waves in the horizontal direction is modeled using gradual damping elements, as discussed in Section 3. The dynamic impedances are calculated per the steps outlined in Section 2; thus, the foundation is modeled as rigid and the mass is set to zero. Calculated impedances for vertical, horizontal and rocking vibration are shown in Figures 6 to 8. As can be seen, the real part of the vertical impedance becomes negative for frequencies in the ranges from 13 to 20Hz and 85 to 95Hz. As mentioned in Section 2, the real and imaginary parts of the dynamic impedance are customarily associated with a frequency dependent spring and dashpot, as shown in Eq. (2) and (3). However, modeling a negative spring is cumbersome in typical structural analysis programs. In this case, the concept of added mass can be used, as illustrated in the following section.

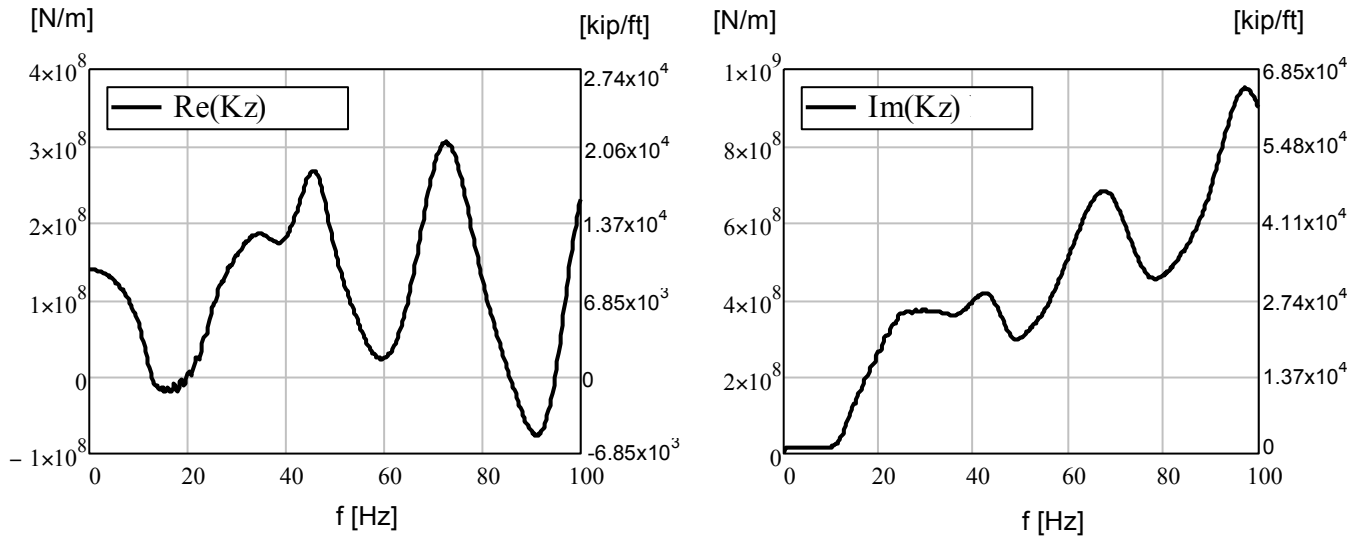


Figure 6—Real and Imaginary parts of the dynamic impedance for vertical vibration.

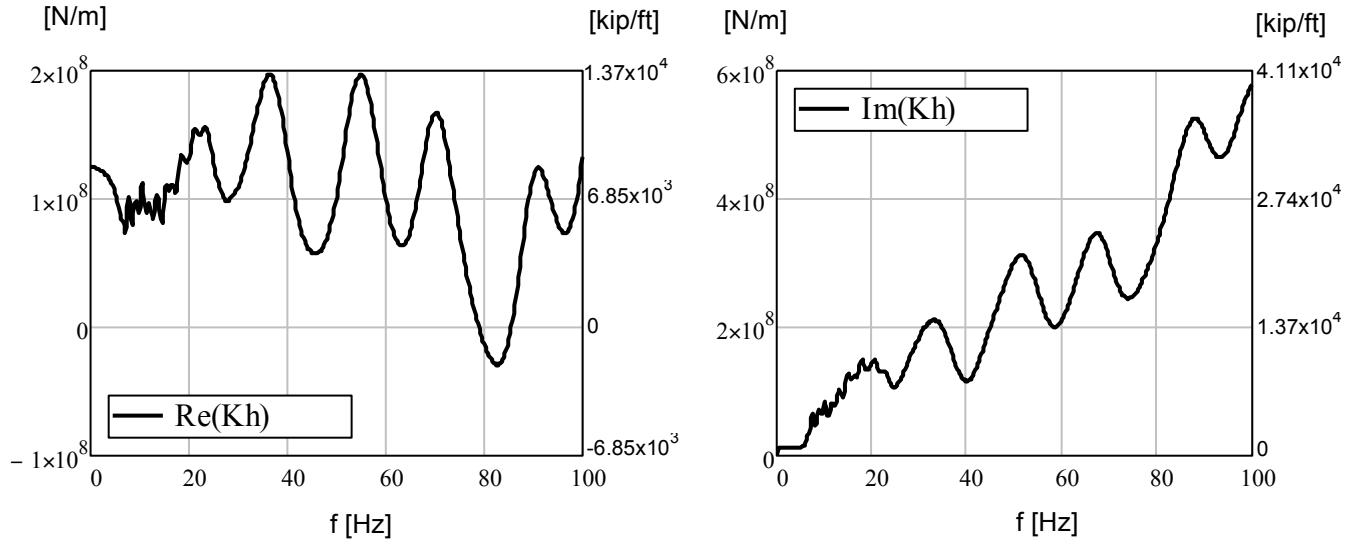


Figure 7—Real and Imaginary parts of the dynamic impedance for horizontal vibration.

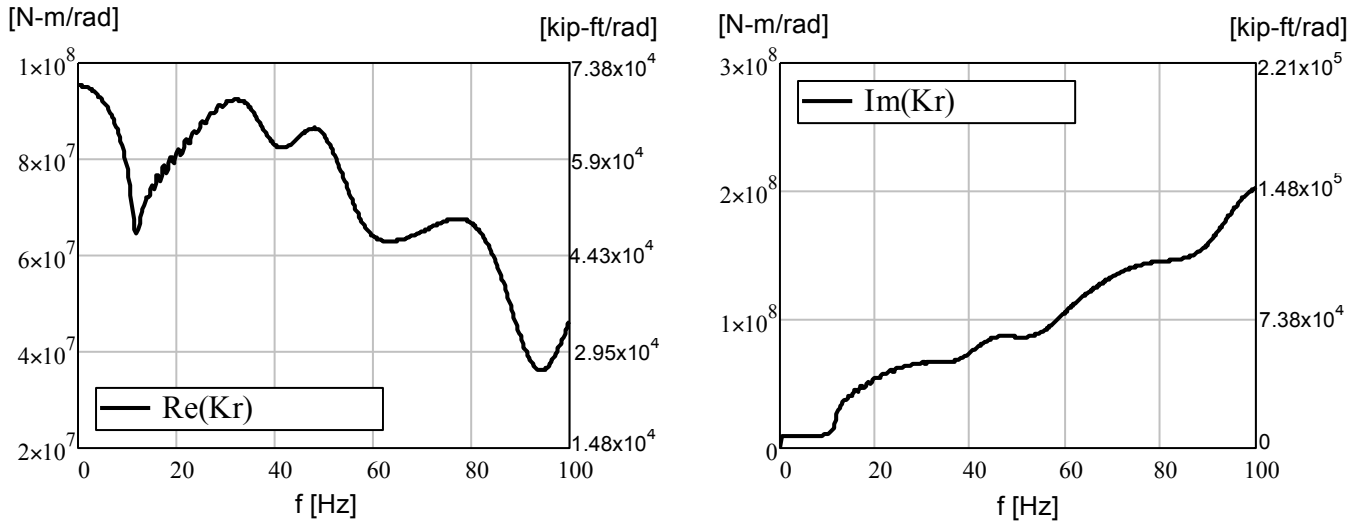


Figure 8—Real and Imaginary parts of the dynamic impedance for rocking vibration.

Block Foundation under Vertical Load

This example illustrates the use of the dynamic impedance and added mass concepts for calculating the response of a machine foundation when the real part (spring coefficient) of the dynamic impedance becomes negative. For this purpose, the foundation from the previous example is analyzed here under vertical vibration. The foundation under consideration has a radius of 1 m (3.28 ft) and its thickness is 0.5 m (1.64 ft), which results in a foundation mass (m) of 3770 kg (8311 lb). The operating speed of the machine (f_m) is 30Hz, the rotating mass $m_r = 500$ kg (1102 lb), and the mass eccentricity $e_r = 1.727\text{E-}4$ m (5.67E-4 ft); which results in a frequency (f) dependent unbalanced load $F_z(t) = (f/f_m)F_0 e^{i\omega t}$, where $F_0 = 3.07$ kN (0.69 kip).

The generic foundation shown in Figure 9 (Ref. [1]) is used to illustrate the degrees of freedom considered while deriving the equation of motion. It must be noticed that the eccentricity between the CG and center of soil

resistance (CR) is zero. Furthermore, the foundation is under a vertical harmonic force $F_z(t)$ acting along the CG, therefore the only displacement experienced by the foundation is $u_z(t)$.

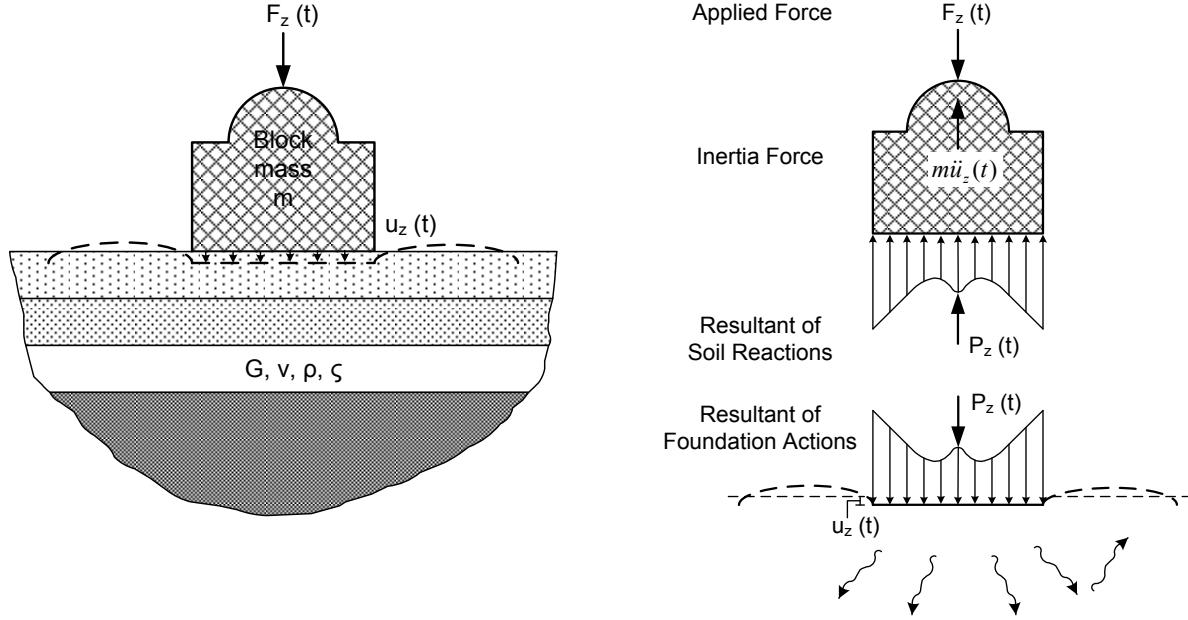


Figure 9—Analysis of the dynamic equilibrium of a vertically vibrating foundation block per Ref. [1].

A detailed derivation of the equation of motion for the foundation depicted in Figure 9 can be found elsewhere, e.g. Ref. [1]. In summary, the dynamic equilibrium equation of the block foundation is:

$$P_z(t) + m\ddot{u}_z(t) = F_z(t) \quad (9)$$

The soil resistance, acting at the CR, is provided by $P_z(t) = K_z u_z(t)$; where K_z is the vertical dynamic impedance, which can be modeled as a frequency dependent spring and dashpot per Eq. (2), i.e., $K_z(\omega) = \bar{k}_z(\omega) + i\omega C_z(\omega)$. Therefore the equation of motion can be rewritten as follows:

$$\begin{aligned} K_z(\omega)u_z(t) + m\ddot{u}_z(t) &= F_z(t) \\ (\bar{k}_z(\omega) + i\omega C_z(\omega))u_z(t) + m\ddot{u}_z(t) &= F_z(t) \end{aligned} \quad (10)$$

For harmonic loading (i.e., $F_z(t) = F_0 e^{i\omega t}$), due to a rotating machine, the solution of above equation is given as shown below, where $u_z(\omega)$ is a complex function.

$$u_z(\omega) = \frac{F_0}{K_z(\omega) - \omega^2 m} = \frac{F_0}{\bar{k}_z(\omega) + i\omega C_z(\omega) - \omega^2 m} \quad (11)$$

As mentioned before, the dynamic spring $\bar{k}_z(\omega)$ shown in Figure 6 becomes negative for frequencies in the range from 13 to 20Hz and 85 to 95Hz. In this case, the machine vibration can be calculated directly using Eq. (11). However, if a standard structural analysis program is used, only static spring coefficients are permitted and negative dynamic springs are not allowed. In this situation, the dynamic spring can be replaced by a spring mass system. Therefore, for $\bar{k}_z(\omega) < 0$, $\bar{k}_z(\omega) = k'_z - \omega^2 m'_s(\omega)$; where k'_z is the static stiffness of the foundation, $k'_z = \bar{k}_z(0)$, and the frequency dependent mass m'_s is calculated as follows:

$$m'_s(\omega) = (k'_z - \bar{k}_z(\omega))/\omega^2 \quad (12)$$

Figure 10 shows the variation of the frequency dependent spring and mass coefficients used to model the vertical stiffness depicted in Figure 6. As can be seen the static stiffness is used where $\bar{k}_z(\omega) < 0$, and the added mass is calculated per Eq. (12). Figure 11 shows the vibration amplitude of the subject foundation as calculated per Eq. (11) using negative spring coefficients or the concept of added mass. As expected the same response is predicted.

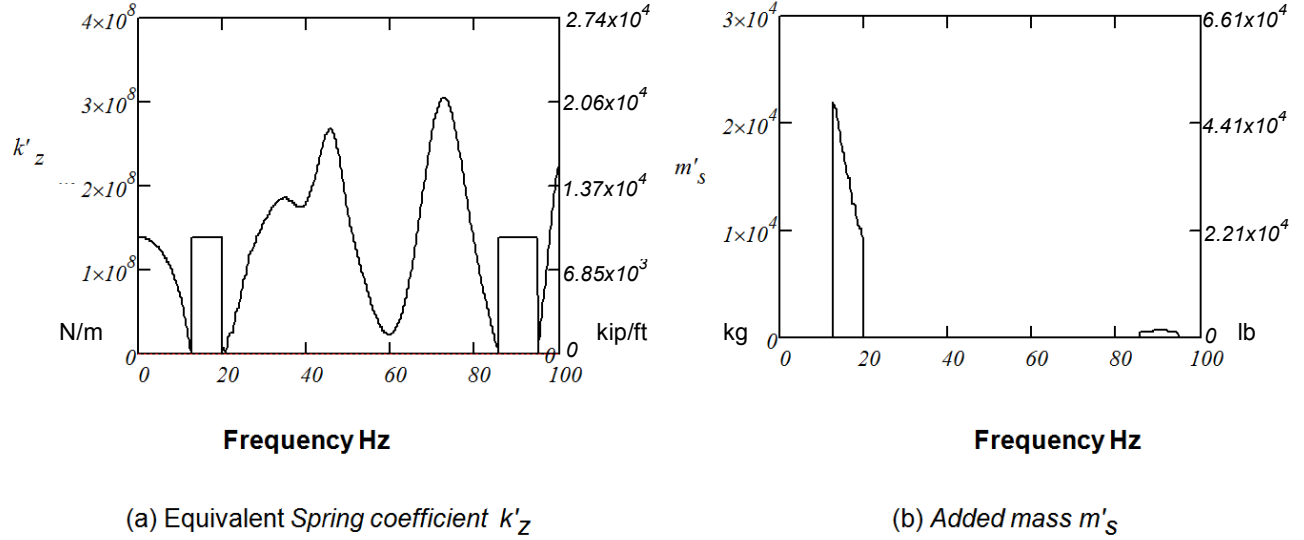


Figure 10—Spring and added mass coefficients for vertical vibration.

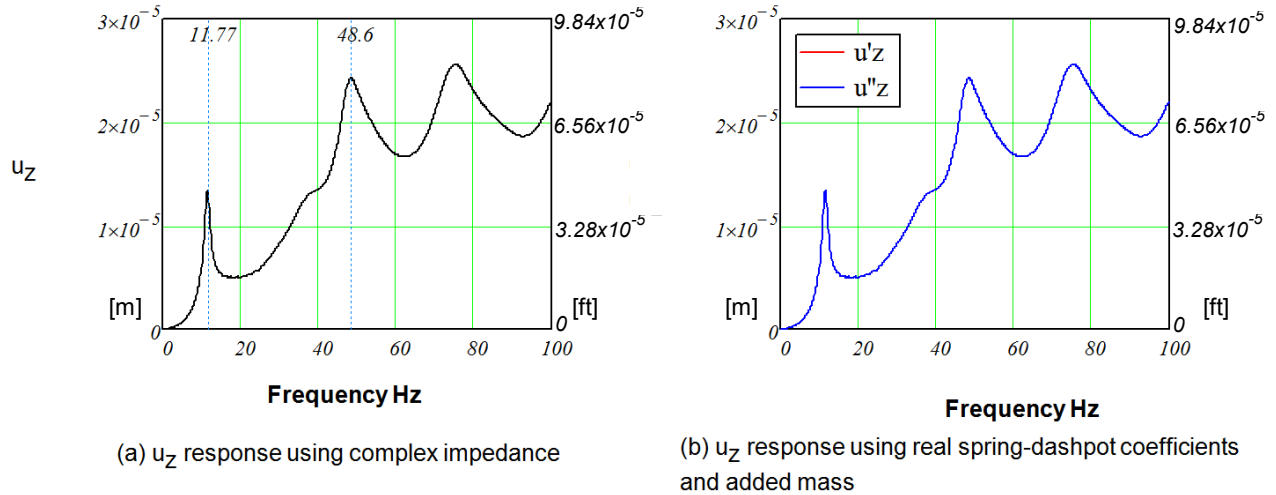


Figure 11—Coast up and coast down response of the rigid block foundation

Block Foundation under Coupled Horizontal and Rocking Vibration

This example illustrates the use of dynamic impedance concept for calculating the response of a machine foundation under coupled horizontal and rocking vibrations. For this purpose, the foundation from the previous example is analyzed under horizontal vibration. The foundation under consideration has a radius of 1 m (3.28 ft) and its thickness is 0.5 m (1.64 ft), which results in a foundation mass (m) of 3770 kg (8311 lb) and a mass moment of inertia $I_0 = 1021 \text{ kg m}^2$ (24229 lb-ft²). The operating speed of the machine (f_m) is 30Hz, the rotating mass $m_r = 500 \text{ kg}$ (1102 lb), and the mass eccentricity $e_r = 1.727\text{E-}4 \text{ m}$ (5.67E-4 ft); which results in a frequency (f) dependent unbalanced load $F_y(t) = (f/f_m)F_0 e^{i\omega t}$ and moment $M_x(t) = (f/f_m)M_0 e^{i\omega t}$, where $F_0 = 3.07\text{kN}$ (0.69 kip) and $M_0 = 0.706 \text{ kNm}$ (0.52 kip-ft). The generic foundation shown in Figure 12 (Ref. [1]) is used to illustrate the degrees of freedom and forces considered while deriving the equation of motion.

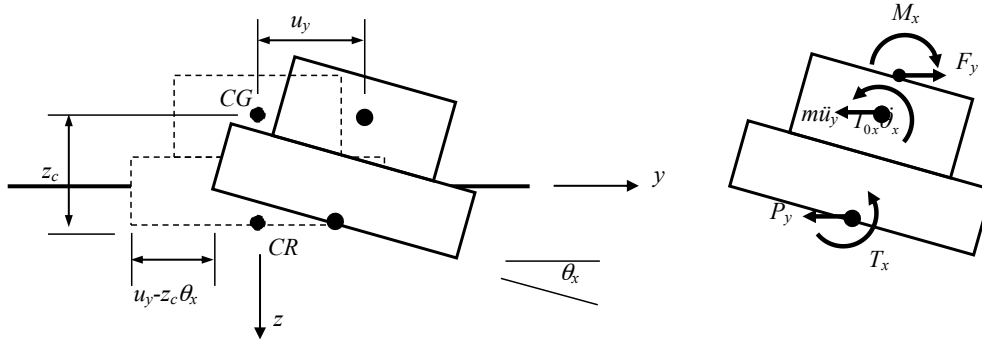


Figure 12—Analysis of the dynamic equilibrium of a horizontally vibrating foundation block per Ref. [1].

A detailed derivation of the equation of motion for the foundation depicted in Figure 12 can be found elsewhere, e.g. Ref. [1]. In summary, the dynamic equilibrium equation of the block foundation is:

$$P_y(t) + m\ddot{u}_y(t) = F_y(t) \quad (13)$$

$$T_x(t) - P_y(t) \cdot z_c + I_{0x}\ddot{\theta}_x(t) = M_x(t) \quad (14)$$

The soil resistance, acting at the CR, is provided by $P_y(t) = K_z(u_y(t) - z_c\theta_x(t)) + K_{yrx}\theta_x(t)$ and $T_x(t) = K_{rx}\theta_x(t) + K_{yrx}(u_y(t) - z_c\theta_x(t))$; where K_y , K_{rx} , K_{yrx} are respectively the horizontal, rocking and coupled dynamic impedances of the subject foundation. Therefore, the equation of motion can be rewritten as follows:

$$K_y(\bar{u}_y - z_c\bar{\theta}_x) + K_{yrx}\bar{\theta}_x - \omega^2 m u_y = F_y \quad (15)$$

$$K_{rx}\bar{\theta}_x + K_{yrx}(\bar{u}_y - z_c\bar{\theta}_x) - [K_y(\bar{u}_y - z_c\bar{\theta}_x) + K_{yrx}\bar{\theta}_x] \cdot z_c - \omega^2 I_{0x}\bar{\theta}_x = M_x \quad (16)$$

Given that the dynamic impedances are complex valued functions of the vibration frequency, above equations can be solved for harmonic loading using complex algebra procedures or standard computer codes for mathematical computations. Figure 13 shows the response of the machine foundation for different vibration frequencies.

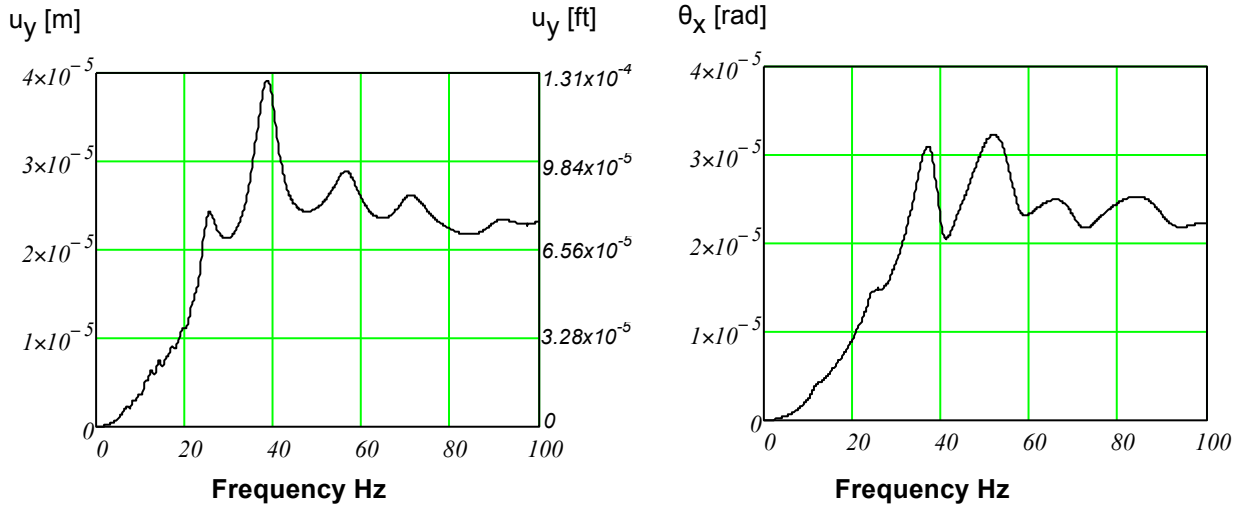


Figure 13—Coast up and coast down response of the rigid block foundation.

5. CONCLUSIONS

The dynamic impedance of a soil supported foundation is highly frequency dependent. This dependency must be accounted for during the design of soil supported machine foundations. FE procedures are proposed in order to predict the dynamic behavior of soil supported foundations. Conclusions derived from the use of FE procedures are as follows:

- Axisymmetric FE models are appropriate to calculate the dynamic impedance of surface, and embedded foundations.
- Gradual damping elements combined with simple boundary elements can be used to properly model the unbounded media in the frequency and time domains. They properly capture the attenuation of stress waves propagating towards infinite.
- The combined use of gradual mesh transitions, gradual damping finite elements and simple boundaries greatly decreases the number of elements required in a FE model, resulting in reduced computational cost without noticeable degradation of the numerical solution.
- The concept of added mass can be used to model negative dynamic stiffness coefficients.

6. REFERENCES

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