PREDICTION OF GROUND MOTION ATTENUATION IN LOW-SEISMICITY REGIONS

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Abstract—*Any quantitative description of earthquake ground motion to be incorporated into a facility's design depends in part on a model that predicts the amplitude of ground shaking as a function, principally, of earthquake size and distance. Models of this type are called strong ground motion attenuation equations or attenuation relationships. The functional form and terms of an attenuation equation must be sufficient to match the main features of the ground motions over the entire expected range of magnitudes, distances, and structural response periods of engineering interest.*

In regions with a history of damaging earthquakes in adequate number and adequately recorded on specialized strong ground motion instruments, the most direct way to develop an appropriate attenuation relationship is to use existing empirical data to fit a representative functional form.

In regions with no history of damaging shaking, it might be supposed that quantitatively precise, defensible estimates of design earthquake ground motions would be of only secondary interest. In fact, the stringent requirements of the design criteria for critical facilities and the relatively short historical record very often require that attenuation relationships be developed even in low-seismicity regions. By definition, however, strong ground motion data are sparse or nonexistent in such regions. Therefore, it is often necessary to use ground motion simulation approaches, such as the stochastic point-source model, to develop a computer-generated, strong ground motion data set that can, in turn, be used to develop an attenuation relationship. The seismological input parameters for the simulations are typically based on regional studies. Given a data set of simulated ground motions and a functional form for the attenuation equation, a maximum likelihood estimation (MLE) procedure may then be used to determine regression coefficients for developing a new ground motion attenuation relationship, much as is done with actual empirical data from seismically active regions.

The strong motion simulation–maximum likelihood regression (SMSIM-MLREG) toolbox described in this paper comprises a set of programs that may be used to simulate ground motions and to develop appropriate ground motion attenuation relationships in a given low-seismicity region based on these motions and the MLE approach. This toolbox provides a streamlined procedure from start to finish for developing a region-specific ground motion attenuation relationship that may be incorporated in a seismic design characterization of a given site.

Keywords—*attenuation relationship, ground motion simulation, maximum likelihood estimation (MLE), maximum likelihood regression (MLREG), nonlinear regression, seismic hazard, source parameter randomization, stochastic point-source model, strong motion simulation (SMSIM)*

Behrooz Tavakoli, PhD

btavakol@bechtel.com

Nick Gregor, PhD

njgregor@bechtel.com

INTRODUCTION

This paper proposes a new methodology for developing ground motion attenuation relationships for given project sites in regions of relatively low seismicity for use as input to seismic hazard analysis. This methodology is most useful for projects that lack a region-specific ground motion attenuation model and/or empirically recorded ground motions.

An appropriate ground motion attenuation model is always one critical component of seismic hazard analysis. Typically, ground motion attenuation relationships have been developed for regions of relatively high historical seismicity both because the empirical strong ground motion data needed to develop empirical attenuation relationships are widely available and because the earthquake damage potential is significant in these high-seismicity regions. However, regions of relatively low seismic activity also may require applicable ground motion attenuation relationships for seismic hazard analysis. Due to the paucity of empirical ground motions for these regions,

ABBREVIATIONS, ACRONYMS, AND TERMS

BLWN	band-limited, Gaussian white noise
ENA	eastern North America
FAS	Fourier amplitude spectrum
LNG	liquefied natural gas
MLE	maximum likelihood estimation
MLREG	maximum likelihood regression
OLS	ordinary least-squares
PGA	peak ground acceleration
PSA	peak spectral acceleration
PSHA	probabilistic seismic hazard analysis
RVT	random vibration theory
SMSIM	strong motion simulation
WNA	western North America

UNITS OF MEASURE AND MATHEMATICAL NOTATIONS

cm	centimeter
f	frequency
f_c	source corner frequency
f_{max}	site corner frequency
Hz	Hertz
km	kilometer
LN	log normal (natural log)
M_0	seismic moment
Q_o	quality factor
R	distance
V_s	shear-wave velocity
$\Delta\sigma$	Brune stress drop (stress parameter)
ε	random error
κ	spectral decay parameter
ω	angular frequency

Ground motion models are needed in areas where there are an insufficient number of strong ground motion recordings from earthquakes.

these attenuation relationships are typically developed based on the numerical modeling of expected ground motions.

Probabilistic seismic hazard analysis (PSHA) has become an important part of earthquake design spectra criteria for projects ranging from conventional structures under modern building codes to critical facilities such as liquefied natural gas (LNG) plants, petroleum facilities, and nuclear power stations. PSHA results depend significantly on the strong ground motion attenuation models (and their uncertainties) incorporated within the analysis. Deterministic ground motion studies are also influenced by the selection of applicable ground motion attenuation models and their associated uncertainties.

The strong motion simulation (SMSIM) approach [1] to generating numerical ground motions, which is based on the stochastic simulation method, uses regionally determined source and propagation path input parameters. The resulting simulations form the synthetic data set that is then used in the regression analysis to develop an applicable region-specific ground motion attenuation model. For the regression analysis, the functional form of the ground motion attenuation model is selected to be relatively simple yet able to capture complicated ground motion behavior in terms of magnitude, distance, and other descriptive parameters. To perform the regression, this paper presents a new maximum likelihood estimation (MLE) procedure—the maximum likelihood regression (MLREG)—and demonstrates its validity and ease of use in developing new empirical ground motion attenuation relationships in regions where no empirical and/or actual strong ground motion data are available.

Taken together, the SMSIM approach and MLREG constitute the SMSIM-MLREG toolbox, a set of programs that may be used to develop attenuation relationships in areas where there are an insufficient number of strong ground motion recordings from earthquakes.

BACKGROUND

To develop an attenuation relationship's functional form, one must know the fundamental characteristics of earthquakes in a given region. Campbell [2] has reviewed and summarized significant factors that can affect strong ground motion attenuation. In general, magnitude, distance, and site conditions are the principal variables used to predict future ground motions. Additional parameters can be incorporated into an attenuation model based on the analysis of the residuals between the data set and the functional model.

Once the functional model has been selected, the next step is to develop a suite of ground motion data for the specific region. With the SMSIM-MLREG toolbox, the data set is developed based on the numerical modeling (i.e., stochastic point-source simulations) of ground motions using seismological input parameters appropriate for a given region. Although more complicated kinematic and dynamic models have been used to simulate ground motions (e.g., see [3]), the stochastic method developed by Boore [1, 4] is used for this analysis because of its simplicity and appropriateness for the development of engineering estimates of strong ground motions.

Having selected a functional form for the attenuation relationship and developed a suite of stochastic ground motion data, one must choose a statistical procedure to determine the period-dependent coefficients in the functional model. Such a procedure is referred to as "regression analysis." If the selected ground motion attenuation model is linear with respect to the coefficients, then standard linear regression procedures can be used. If not, a nonlinear regression procedure must be used. The MLREG program developed in this study uses the statistical toolbox in the MATLAB®[1] environment to determine regression coefficients for either linear or nonlinear equations.

GROUND MOTION ATTENUATION RELATIONSHIPS

A basic form for an attenuation relationship is the logarithmic equation

$$\ln Y = c_1 + c_2 M - c_3 \ln[f_1(R)] - c_4 f_2(R) + c_5 f_3(P) + \varepsilon, \tag{1}$$

where Y is the ground motion parameter of interest, such as peak ground acceleration (PGA) or peak spectral acceleration (PSA) at some defined period; M is magnitude; R is a measure of the distance from the source to the site being considered; P is a description of the local conditions beneath the site; and ε is a random error term with a mean of zero and a standard deviation of $\sigma_{\ln Y}$, representing the uncertainty in Y. In some studies an additional term to account for the observed differences in ground motions due to earthquake mechanism (e.g., strike-slip, thrust, normal) is included in the mathematical representation of the ground motion parameter. Several more-complex ground motion attenuation models are currently in use as well to model empirical ground motion data. The period-dependent coefficient parameters c_1 through c_5 for this model are obtained from the regression process for spectral period T. In more complicated forms of Equation 1, the coefficients c_2, c_3, c_4, and c_5 can be defined in terms of M and R.

The physical basis for this simplified attenuation relationship is seismological theory. In Equation 1, the first term, $c_1 + c_2M$, is called the magnitude scaling term, and is consistent with the original definition of earthquake magnitude, in which ground motions increase exponentially with a linear increase in magnitude. Alternative functional forms can be used for the magnitude scaling term. For example, the piecewise linear relationships (e.g., see [5]) or the quadratic magnitude term (e.g., see [6]) have been proposed where magnitude saturation is an issue for $M \geq 6.0$. Magnitude saturation occurs when incremental increases in magnitude are associated with ever-smaller increases in ground motion of a defined period. In unconstrained and extreme cases, this can lead to the unrealistic effect of predicting, for a given distance and on firm foundations, smaller ground motions with increasing magnitude. In some cases, a purely statistical regression on a given empirical data set can lead to a ground motion attenuation model that contains this unacceptable magnitude saturation result.

Simplified attenuation models can be based on seismological theory.

The terms $-c_3\ln[f_1(R)]$ and $-c_4 f_2(R)$ are called distance scaling terms. They are consistent, respectively, with the geometrical spreading attenuation of seismic waves as they propagate away from the earthquake source and with anelastic attenuation (i.e., material damping and scattering). Both of these functional terms are associated with the basic seismological principles of seismic wave propagation in elastic media. In some cases, these terms are varied as a function of magnitude to accommodate the observed distance saturation properties of strong ground motion.

The last term, $+c_5 f_3(P)$, is used to model the effect of the local site conditions on the ground motions. The coefficient c_5 can be a function of magnitude and distance if $f_3(P)$ is found to correlate with these parameters. For the SMSIM-MLREG toolbox presented in this study, simulated ground motions are all for the same site conditions; hence, this term is not used in the regression analysis.

Random error term ε is usually assumed to be log-normally distributed. The error term is the difference between a ground motion observation (based on either numerical simulations or empirical data) and its predicted value. Although this total uncertainty is typically separated into

[1] MATLAB is a high-level language and interactive environment that enables computationally intensive tasks to be performed faster than with the more traditional programming languages.

intra- (within) and inter- (between) earthquake components, the MLREG program currently only computes the total uncertainty between the data set and functional model.

In general, the selection of a functional form for the ground motion attenuation relationship should be guided by the data set. If the data set represents relatively long-distance ground motions (i.e., distances greater than 100–200 km), then the distance saturation (and to a lesser extent the magnitude saturation discussed earlier in this section) can be ignored and a simple functional model (i.e., a linear functional model) is justified. If the data set represents a relatively uniform subset of data, then a functional form having only a few parameters is appropriate.

Stochastic ground motion modeling is an acceptable methodology for the simulation of strong ground motions.

STOCHASTIC GROUND MOTION MODEL

The stochastic method assumes that the average horizontal component of ground motion may be modeled as band-limited, Gaussian white noise (BLWN) and that the peak amplitude may be approximated using random vibration theory (RVT). [7, 8, 9, 10] This method assumes that the seismic shear-wave energy represented by the Fourier amplitude spectrum (FAS) is band-limited by the source corner frequency (f_c) at low frequencies and by the site corner frequency (f_{max}) or the spectral decay parameter (κ) at high frequencies. This section gives a brief overview of the seismological models used in the stochastic ground motion model and, in turn, in the SMSIM program.

Fourier Amplitude Spectrum

A point-source stochastic model in the frequency domain assumes that the total FAS of acceleration $A(f)$ for horizontal ground motions due to shear waves may be modeled by the general relation [1]

$$A(f) = E(f, M_0)^* D(f, R)^* P(f), \qquad (2)$$

where M_0 is the seismic moment (dyne-cm), R is the distance (km), and f is the frequency (Hz). $E(f, M_0)$ is the point-source spectrum term, $D(f, R)$ is a diminution factor accounting for both geometrical and anelastic attenuation, and $P(f)$ is a low-pass filter to model the decrease of Fourier amplitude spectra at high frequencies (i.e., site amplification factor).

Point-Source Spectrum Model [$E(f, M_0)$]

The most commonly used point-source model is based on the Brune spectrum. [11, 12] This basic seismological model of a ground acceleration spectrum has a simple ω^{-2} shape, where ω is angular frequency (that is, $2\pi f$). This model assumes that the earthquake source is a circular fault and that the ground acceleration spectrum from this simplified source has a ω^{-2} decay for frequencies below source corner frequency f_c and is flat for frequencies greater than f_c but less than site corner frequency f_{max}. The amplitude spectrum level begins to drop at higher frequencies beyond f_{max}. The choices of f_c and f_{max} depend mainly on the earthquake size and the site condition, respectively.

The Brune stress drop ($\Delta\sigma$) is generally computed from the high-frequency energy of the Fourier amplitude spectra of measured earthquakes. [13] Higher stress drops increase the corner frequencies. Higher corner frequencies in turn increase the amplitude levels at higher frequencies. The stress drop value may be used as a fitting parameter to adjust the source spectrum model to adequately model observed ground motions that may not fit a single-corner source model. To avoid confusion, some seismologists prefer to call the Brune stress drop the "stress parameter."

Filter Function of the Transfer Media [$D(f, R)$]

The loss of wave energy within a geological medium (crustal attenuation) is calculated by multiplying a point-source geometrical attenuation factor by a deep crustal damping factor. The geometrical attenuation factor is modeled using the distance parameter and depends mainly on the regional thickness of the Earth's crust. For example, in eastern North America (ENA), the geometric attenuation of seismic waves may be given by a three-part expression (e.g., see [14]). The spherical spreading of body waves results in an R^{-1} amplitude decay within a 70 km range. Beyond 70 km, the direct shear waves are superimposed on waves reflected from the Moho[2], offsetting any decay in the amplitude of seismic waves between 70 and 130 km (i.e., R^0 amplitude decay). The cylindrical spreading of surface waves results in an $R^{-0.5}$ amplitude decay beyond 130 km. In western North America (WNA), an example of the geometrical attenuation model is defined as spherical spreading of R^{-1} to a distance of 40 km and a cylindrical spreading of $R^{-0.5}$ beyond 40 km. [15] Note that there are many other published geometrical attenuation models for both WNA and ENA. Any geometrical model

[2] The Mohorovičić discontinuity, first identified in 1909 by Andrija Mohorovičić, a Croatian seismologist and usually referred to simply as the Moho, is the boundary between the Earth's crust and mantle.

can be used with the SMSIM-MLREG toolbox to develop ground motion attenuation relationships for a given region.

The shallow crustal damping (diminution of ground motions) may be modeled as being proportional to the factor $\exp(-\gamma R)$, where R is distance and γ is the coefficient of anelastic attenuation, given by Campbell [16] as

$$\gamma = \frac{\pi f}{Q V_s}, \tag{3}$$

where quality factor Q models anelastic attenuation and scattering within the deep crustal structure and V_s is the seismic shear-wave velocity used to determine Q. The quality factor model is considered as a function of frequency and may be modeled as the median of seismic wave attenuation within the lower and upper uncertainty levels. [14, 17]

Caution should be used in coupling a given geometrical attenuation model and an anelastic attenuation model because these two models are highly correlated.

Filter Function of the Local Site Conditions [*P*(*f*)]

Anderson and Hough [18] proposed a low-pass filter based on the spectral decay parameter (κ), which produces a near surface attenuation of high frequency energy. This κ-filter (shallow crustal damping) is defined as the high-frequency slope of the Fourier amplitude spectra. Anderson and Hough [18] found κ approaching a constant value near the epicenter of a seismic event and assumed that it is dependent on the subsurface geology. At larger distances from the source, κ increases slightly due to path effects associated with wave propagation in the crust and quality factor Q_o.

The shallow crustal model beneath the site defines shear-wave velocity V_s and density as functions of depth. When seismic waves travel through the crust, the amplitude, frequency content, and duration of ground surface motions change. The extent of these changes depends mainly on the geometry and properties of the subsurface materials. Site amplification factors may be computed using the quarter-wavelength approximation method. [19] In this method, site amplification factors at a specific frequency (or wavelength) are given by the square root of the ratio between the seismic impedance (product of V_s and density) at the site averaged over a depth equal to one quarter of the wavelength and the seismic impedance at the source.

Peak Ground Motion Parameters [ln *Y*]

The stochastic model can provide the average amplitude level of earthquakes for a wide range of magnitudes and distances. [1, 10] First, the stochastic model is used to generate an acceleration time history as Gaussian white noise. Then, the FAS of the time history is combined with the seismological model of ground motion to obtain the desired spectrum shape at the near-source distance as a function of earthquake size. Finally, RVT is used to determine maximum ground motion parameters, such as PGA and PSA, from root mean square parameters. [20] These simulation ground motions are used to develop a new attenuation relationship.

The stochastic model is used to simulate PGA as well as spectral acceleration ground motion values.

The ground motion values of Y from Equation 1 are obtained by the Cartwright and Longuet-Higgins [21] approach using the maxima of a random function. This approach assumes that the phases of a stochastic function are random and uniformly distributed between 0 and 2π. The ground motion duration is given by $T_{gm} = T_s + T_p$, in which T_s is the source duration and T_p is the path duration. The source duration can be defined as the time for the fault to rupture and is proportional to the inverse of the source corner frequency. [10] The path duration depends on the epicentral distance and can be estimated based on the method proposed by Atkinson and Boore. [14]

MAXIMUM LIKELIHOOD ESTIMATION METHOD

With a proposed functional form with which to model the attenuation of strong motion and a body of strong motion "data" simulated using principles of wave generation and propagation appropriate to a region of interest, the final task becomes fitting the data to the model.

Equation 1 can be recast in the form of a general attenuation relationship,

$$y_i = f(x_i; \beta) + e_i, \tag{4}$$

where y_i is a ground motion parameter, such as PGA or PSA, from the i^{th} earthquake and x_i is a vector of predictors such as earthquake magnitude, source-to-site distance, or other site and source conditions, also from the i^{th} earthquake. The β in this equation is a vector of regression coefficients to be estimated, and f is a known functional form. Error term e_i is assumed to be an independent zero-mean normal random variable with a constant variance, σ^2.

The non-linear regression is approximated as a linear regression based on a Taylor expansion.

Initially, the functional form is assumed to be linear with respect to the coefficients to be estimated. (As is discussed later, we can modify the linear procedure using a Taylor expansion to accommodate a nonlinear regression model.) For example, a suitable transformation of data can be found to reduce a nonlinear attenuation model to a linear attenuation model. Equation 4 can now be replaced by a linear system:

$$y_i = x_i \beta + e_i. \tag{5}$$

Given the assumption that e_i in Equation 5 is normally distributed, then dependent variable y_i is normally distributed as well. This implies that y_i is a continuous and unbounded normal distribution function. Suppose that the data set consists of N ground motion records coming from E earthquakes (x_i, y_i), $i = 1, 2,\ldots, N$. It follows that the logarithm of the likelihood function for the whole data set is given in matrix form by the following expression:

$$1\text{n}\,\ell(y \mid x,\beta,\sigma^2) = -\frac{N}{2}\,1\text{n}(2\pi\sigma^2) - \frac{1}{2\sigma^2}(y - X\beta)^T(y - X\beta), \tag{6}$$

where T denotes matrix transposition. The maximum likelihood estimates of the parameters (β, σ^2) are found by maximizing the log-likelihood function above. Finding the values of β and σ^2 that maximize the log-likelihood is easily done by taking the derivative of Equation 6 with respect to β and σ^2, setting it equal to zero, and solving for the parameters to be estimated.

Following this procedure, we can find a closed-form solution for β by the following matrix expression:

$$\hat{\beta} = (X^T X)^{-1} X^T y. \tag{7}$$

Equation 7 indicates that MLE and ordinary least-squares (OLS) both give the same estimator for the regression coefficients. Expected or predicted ground motion $\hat{y}$ is calculated by substituting $\hat{\beta}$ into Equation 4. Thus, we have the prediction of ground motion values by the expression

$$\hat{y} = X\hat{\beta} = X(X^T X)^{-1} X^T y = Hy, \tag{8}$$

where $H = X(X^T X)^{-1} X^T$ is called the "hat" matrix because it transforms or projects observed ground motion y into predicted ground motion $\hat{y}$.

Because we have already obtained $\hat{\beta}$, we can find a closed-form solution for σ^2 by taking the derivative of Equation 6 with respect to σ^2, setting the result equal to zero, replacing β with its estimate $\hat{\beta}$, and solving for σ^2. The solution leads to the following matrix expression:

$$\hat{\sigma}^2 = \frac{(y - X\hat{\beta})^T (y - X\hat{\beta})}{N} \tag{9}$$

To accommodate a nonlinear regression model, we modify the above procedure by using a Taylor expansion. We begin by first noting that in a small neighborhood of $\hat{\beta}$, the true value of β can be found using a truncated Taylor expansion. Thus, the general attenuation relationship in Equation 4 can be expressed as the following matrix form:

$$y = f(X,\hat{\beta}) - X^*\hat{\beta} + X^*\beta + \varepsilon, \tag{10}$$

where X^* is the partial derivative matrix X with respect to β, and $\hat{\beta}$ is a trial value of β.

We can obtain an approximate linear attenuation model by the expression

$$y^* = X^*\beta + \varepsilon, \tag{11}$$

where $y^* = X^*\hat{\beta} + [y - f(X,\hat{\beta})]$, X^* is the $N \times p$ partial derivative matrix X, and p is the number of regression coefficients to be estimated.

Therefore, the process of maximizing the likelihood in Equation 6 involves calculating matrix X^* and vector y^* and then replacing them with X and y, respectively. There is no closed-form solution for Equation 11 because X^* is now a function of β. Thus, in most cases, an iterative method must be employed to obtain β and σ^2.

We employed the Gauss-Newton algorithm to solve this problem. The proposed computation algorithm used in the MLREG program is described as follows:

1. Start with initial estimates for β.
2. Estimate $\hat{\beta}$ in attenuation model Equation 7 by substituting X into X^* and y into y^*.
3. Treat the $\hat{\beta}$ value as the initial value in the next approximated linear model.
4. Repeat steps 1–3 until the solution converges, that is, for successive iterations j, $j + 1$:

$$\left| \frac{\hat{\beta}_{j+1} - \hat{\beta}_j}{\hat{\beta}_j} \right| < \delta, \tag{12}$$

where δ is a predetermined small amount (i.e., 1.0×10^{-6}) in the MLREG program.

5. The value of σ^2 given by Equation 9 is not unbiased. An unbiased estimate is

$$\hat{\sigma}^2 = \frac{(y - X\hat{\beta})^T (y - X\hat{\beta})}{N - p} \, . \qquad (13)$$

RANDOMIZATION OF SOURCE PARAMETERS

In the development of the ground motion simulations, certain seismological input parameters for the stochastic point-source are typically defined based on a median value with an associated uncertainty. To capture this parametric uncertainty, the simulation procedure can generate suites of ground motions based on the defined median and uncertainty values for any or all input parameters. As part of the SMSIM-MLREG toolbox, a streamlined preprocessing program was created that allows the user to easily generate suites of necessary input parameter files based on the user-defined median parameter values and their associated uncertainties.

As an example, the SMSIM MLREG toolbox was used to generate a suite of 100 randomized input cases based on the parameters given in **Table 1**. The randomization parameters are given for the four input parameters: quality factor (Q_o), stress parameter ($\Delta\sigma$), spectral decay parameter (κ), and hypocentral depth. The resulting statistical values based on the randomized 100 realizations for these four input parameters are also listed in Table 1. In comparing the computed median and uncertainty values with the user-defined values listed in Table 1, the randomized selection of each of the four input parameters is considered acceptable. The statistical distribution of the 100 randomized parameter values for Q_o, $\Delta\sigma$, κ, and hypocentral depth are plotted in **Figures 1a, 1b, 1c, and 1d**, respectively, along with the expected log-normal distribution of each parameter, given the median and uncertainty values listed in Table 1. The randomized values are shown by the red bars, and the expected values are shown by the blue bars. The results shown in Figures 1a–1d and the statistical results listed in Table 1 confirm the acceptability of the randomized selection procedure.

Note that exact agreement between the numerical median and uncertainty values and the graphical distribution plots shown in Figures 1a–1d is not expected based on the limited number of realizations (i.e., 100 samples). For this example, the minimum and maximum values for three parameters—Q_o, $\Delta\sigma$, and κ—are selected with a large enough range so that the randomized values would not be expected to fall outside these limits. For the hypocentral depth parameter, the realistic upper and lower depth range values of 5.0 and 20.0 km, respectively, limit the observed distribution of randomly selected hypocentral depth values. In addition, this limited upper and lower range leads to the smaller-than-expected uncertainty value of 0.362 when compared to the user-defined uncertainty value of 0.60 and a skewed distribution of hypocentral values greater than the given input median value of 8.0 km (see Figure 1d). Increasing these upper and lower limits for the hypocentral depth would result in a better agreement between the input values and the resulting statistical values from the 100 random samples. However, this would lead to unrealistic seismological values for the distribution of hypocentral depths.

For the simulation of ground motions, specific seismological input parameters can be randomized.

Table 1. User-Defined Input Parameters (Median and Uncertainty Values) and Corresponding Output Statistical Values for Validation Example with 100 Realizations

Parameter	Q_o	$\Delta\sigma$	κ	Hypocentral Depth, km
Base Median	351	120	0.006	8.0
Base Uncertainty (Sigma)	0.4	0.7	0.3	0.6
Computed Median	351.05	120.88	0.0061	9.97
Computed Sigma	0.310	0.648	0.304	0.362
Number of Values	100	100	100	100
Minimum Allowable Value	100.00	14.00	0.0024	5.0
Minimum Value	168.19	24.02	0.0026	5.01
Maximum Allowable Value	1,200.00	980.00	0.0150	20.0
Maximum Value	680.15	553.41	0.0117	19.22

The newly developed toolbox allows for the randomization of parameters based on median and uncertainty values from seismological studies.

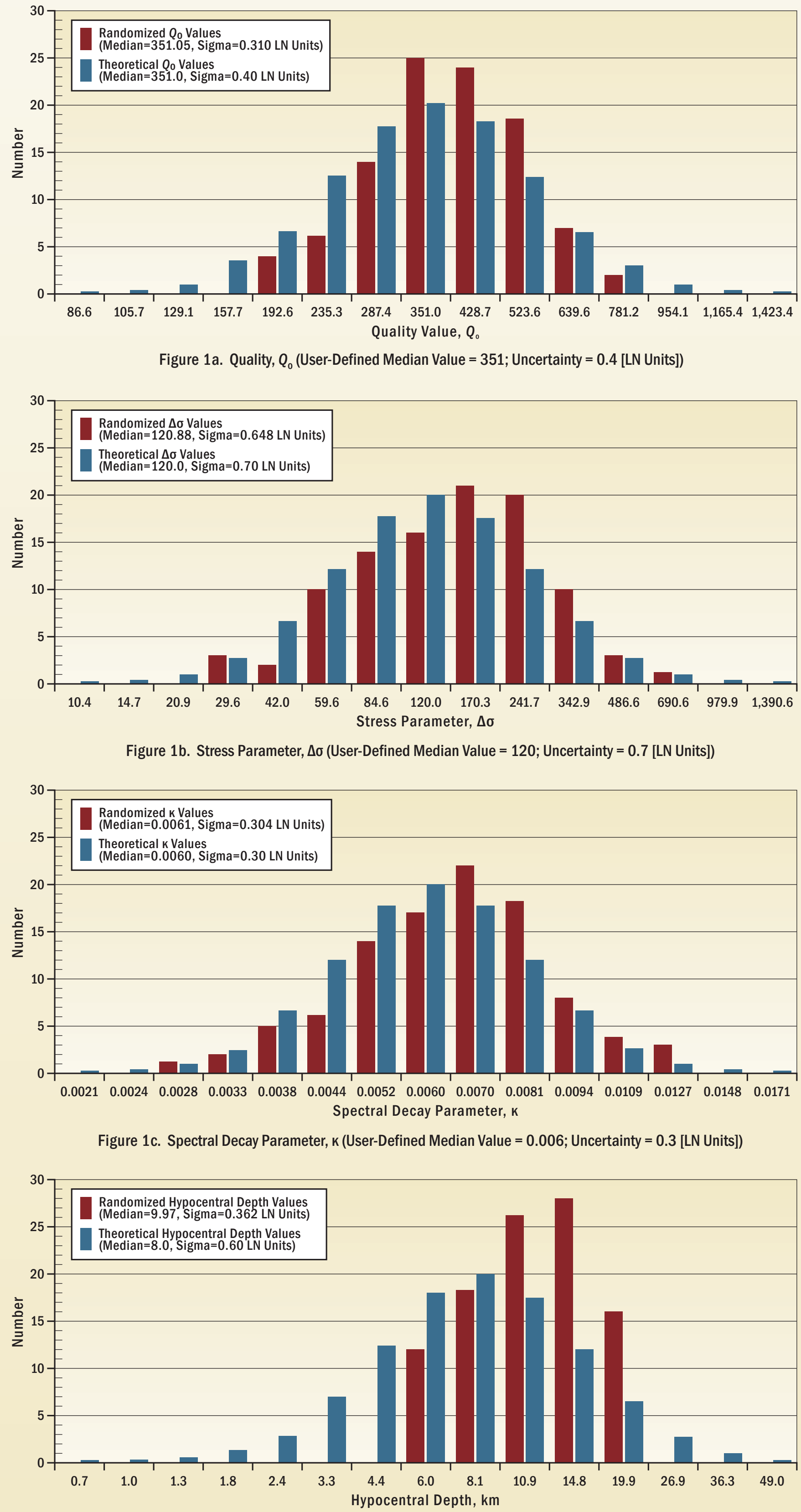

Figure 1a. Quality, Q_o (User-Defined Median Value = 351; Uncertainty = 0.4 [LN Units])

Figure 1b. Stress Parameter, Δσ (User-Defined Median Value = 120; Uncertainty = 0.7 [LN Units])

Figure 1c. Spectral Decay Parameter, κ (User-Defined Median Value = 0.006; Uncertainty = 0.3 [LN Units])

Figure 1d. Hypocentral Depth Values, km (User-Defined Median Value = 8.0; Uncertainty = 0.6 [LN Units])

Figure 1. Histogram Distributions of Seismological Input Parameters for 100 Randomized Realizations (Red Bars) and the Expected Distribution Values (Blue Bars) Based on the User-Defined Median Values and Associated Uncertainties

REGION-SPECIFIC GROUND MOTION ATTENUATION MODEL

In this example, the SMSIM-MLREG toolbox was used to perform nonlinear regression on a simulated ground motion data set to determine regression coefficients for the median prediction ground motion equations and the variability about the median. Two nonlinear ground motion attenuation models, specified below, were considered for fit to a new simulated data set of ground motions. The sample regional ground motion attenuation model presented is based on the programs and methodology introduced in the earlier sections. This sample regional ground motion attenuation model is developed for illustrative purposes and is not representative of any particular site.

Most regional ground motion input parameters for this practical example were taken from Silva et al. [22] Stochastic ground motions were generated based on the Brune single-corner source model for the suite of magnitudes 4.5, 5.5, 6.5, and 7.5 and distances 1, 5, 10, 15, 20, 30, 50, 75, 100, 150, 200, 300, and 500 km. For each magnitude and distance value, the program was used to randomly select 30 realizations, given the median and uncertainty values for Q_o, $\Delta\sigma$, κ, and hypocentral depth listed in Table 2. In addition to the median and uncertainty values for the four randomized parameters, the minimum and maximum values are also listed in **Table 2**. These bounding values were selected to be large enough to span the range of expected parameter values except for the hypocentral depth parameter, which is limited by the observed range in hypocentral depths. The resulting statistical median and uncertainty values for the four selected input parameters are listed in **Table 3**.

A simple region-specific example shows the benefit of the newly developed toolbox.

Table 2. Input Parameter Values for Sample Regional Ground Motion Attenuation Model Case

Parameter	Median/Base Value	Sigma (LN Units)	Reference
Quality Factor (Q_o)	351 (Min. = 100; Max. = 10,000)	0.40	Mid Continent see [20, 22]
Stress Parameter ($\Delta\sigma$)	120.0 (Min. = 10; Max. = 750)	0.70	see [20, 22]
Spectral Decay Parameter (κ)	0.006 (Min. = 0.0001; Max. = 0.100)	0.30	see [17, 20]
Hypocentral Depth (km)	0.84	–	Mid Continent see [20, 22]
Crustal Amplification Factors	ENA Hard Rock	–	see [23]
Path Duration	Atkinson and Boore [14] model with hinge points at 75 and 100 km	–	see [14]
Shear-Wave Velocity (V_s) at Source	3.6 km/sec	–	see [23]
Density	2.8 g/cm^3	–	see [23]
Geometrical Spreading	$R^{-(1.0296-0.0422*(M-6.5))}$ for R ≤80 $R^{-(1.0296-0.0422*(M-6.5))/2}$ for R >80	–	see [22]

Table 3. Statistical Results for the Four Selected Input Parameters Based on Random Sampling of 30 Realizations

Parameters/Magnitudes	Median/Base Value	Sigma (LN Units)	Median Statistical Value	Sigma (LN Units)
Quality Factor (Q_o)				
M4.5 M5.5 M6.5 M7.5	351	0.40	347.49 347.49 356.63 369.46	0.391 0.391 0.301 0.350
Stress Parameter ($\Delta\sigma$)				
M4.5 M5.5 M6.5 M7.5	120.0	0.70	128.0 128.0 130.3 149.0	0.635 0.635 0.629 0.649
Spectral Decay Parameter (κ)				
M4.5 M5.5 M6.5 M7.5	0.006	0.30	0.0061 0.0061 0.0064 0.0057	0.267 0.267 0.318 0.246
Hypocentral Depth (km)				
M4.5 M5.5 M6.5 M7.5	6.0 6.0 8.0 10.0	0.6 0.6 0.6 0.6	5.93 5.93 9.08 10.79	0.455 0.455 0.406 0.397

Creating 30 realizations each for the complete set of four magnitudes and 13 distances produced a total of 1,560 input files. Next, the toolbox was used to generate ground motions for each of the 1,560 input cases at the following seven frequencies: 100, 25, 10, 5, 2.5, 1, and 0.5 Hz. This resulted in a total of 1,560 summary case files of ground motion estimates for each realization. Finally, the ground motions from each of the 1,560 case files were compiled into a single ground motion data set file. Given this sample data set, the MLREG program within the SMSIM-MLREG toolbox was implemented to fit the data set to the following two ground motion attenuation functional forms:

Example ground motion model would be available for seismic hazard studies.

Model 1: $\ln Y = C_1 + C_2{*}M + C_3{*}(M - 6)^2 + (C_4 + C_5M){*}\ln(R + e^{C6}) + (C_7 + C_8M){*}R$ and

Model 2: $\ln Y = C_1 + C_2{*}M + C_3{*}(M - 6)^2 + (C_4 + C_5M){*}\ln(R + e^{C6})$,

where R is the epicentral distance (i.e., Joyner-Boore distance for this case, in which the ground motions are simulated as point sources).

The $N \times p$ matrix X^* in Equation 10 is defined by the $N \times p$ derivative matrix X with respect to the regression coefficients, where N is total number of data points and p is total number of coefficients. Note that X^* now is a function of the regression coefficients (i.e., it is nonlinear) and, hence, there is a need to employ an iterative algorithm. The regression coefficients found for the seven spectral frequencies are listed in **Table 4** for both ground motion attenuation models.

The comparison between the simulated data and the two regression models for 100 Hz is plotted in **Figures 2a and 2b** for the two moment magnitude values 5.5 and 7.5. Both ground motion attenuation models show an acceptable fit to the data set and predict similar median ground motions. The companion results for 1 Hz are shown in **Figures 3a and 3b**. Similar results are observed for the same two magnitude values.

CONCLUSIONS

The key feature and product of this investigation is a new streamlined method to develop an applicable ground motion attenuation relationship for a given project site that can be used in seismic hazard analysis. This methodology is most useful for projects that lack a region-specific ground motion attenuation model and/or empirically recorded ground motions.

Table 4. Regression Coefficients for Both Model 1 and Model 2 Fits to Dataset

MODEL 1							
Coefficient	**100 Hz**	**25 Hz**	**10 Hz**	**5 Hz**	**2.5 Hz**	**1 Hz**	**0.5 Hz**
C_1	3.8726	3.8741	2.8006	1.1903	–1.3238	–5.7465	–8.5339
C_2	–0.0281	0.0403	0.1661	0.3493	0.659	1.174	1.4056
C_3	–0.0054	–0.0176	–0.0401	–0.0781	–0.1645	–0.3286	–0.3584
C_4	–3.2738	–3.1258	–2.9472	–2.7335	–2.45	–2.1858	–2.3746
C_5	0.3014	0.2824	0.2553	0.223	0.1819	0.1541	0.1971
C_6	2.1127	2.1194	2.1642	2.216	2.2672	2.2032	2.0164
C_7	0.0048	0.0046	0.0043	0.0037	0.0029	0.0026	0.0042
C_8	–0.001	–0.001	–0.0009	–0.0007	–0.0006	–0.0005	–0.0007
Sigma	0.548	0.5096	0.4953	0.4789	0.451	0.4037	0.3662
MODEL 2							
Coefficient	**100 Hz**	**25 Hz**	**10 Hz**	**5 Hz**	**2.5 Hz**	**1 Hz**	**0.5 Hz**
C_1	3.3958	3.4084	2.0367	0.3721	–2.0361	–6.4266	–9.7318
C_2	0.2317	0.2895	0.4059	0.5621	0.829	1.3234	1.6194
C_3	–0.0054	–0.0176	–0.0401	–0.0781	–0.1645	-0.3286	–0.3584
C_4	–2.9594	–2.8259	–2.6063	–2.4096	–2.184	–1.9387	–1.9616
C_5	0.2049	0.1905	0.1708	0.1503	0.125	0.104	0.1245
C_6	2.449	2.4475	2.3856	2.3688	2.3727	2.2873	2.0495
Sigma	0.5577	0.5189	0.5012	0.4826	0.453	0.4053	0.3703

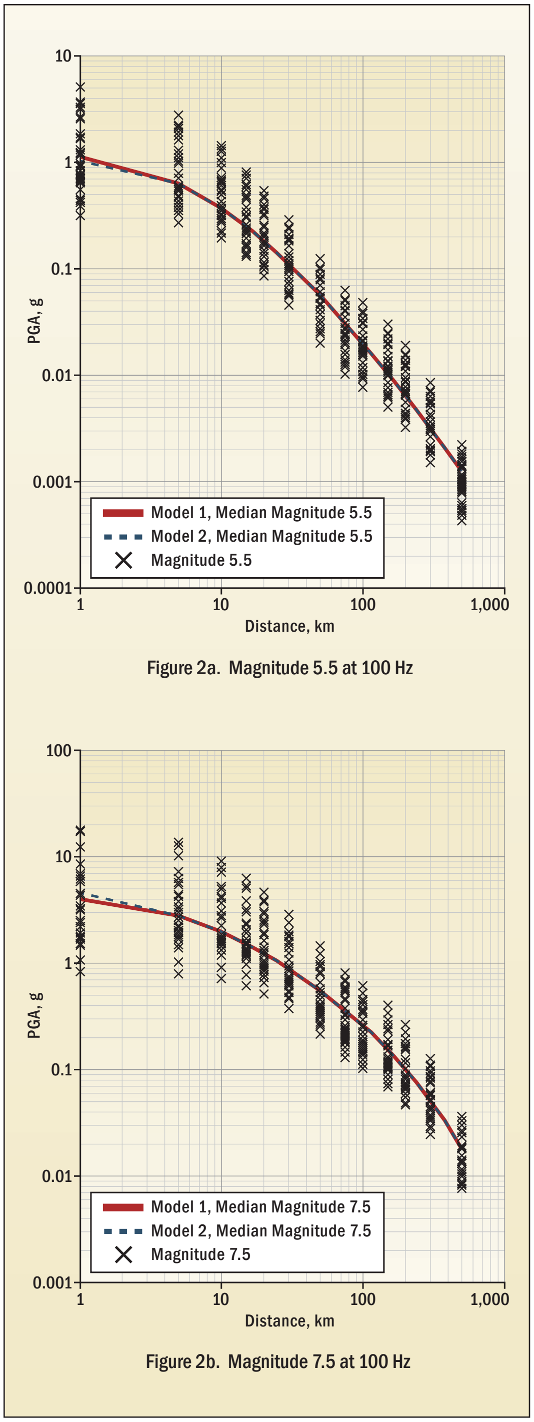

Figure 2a. Magnitude 5.5 at 100 Hz

Figure 2b. Magnitude 7.5 at 100 Hz

Figure 2. Comparisons Between Simulated Ground Motions and Two Regression Model Prediction Curves, as a Function of Epicentral Distance

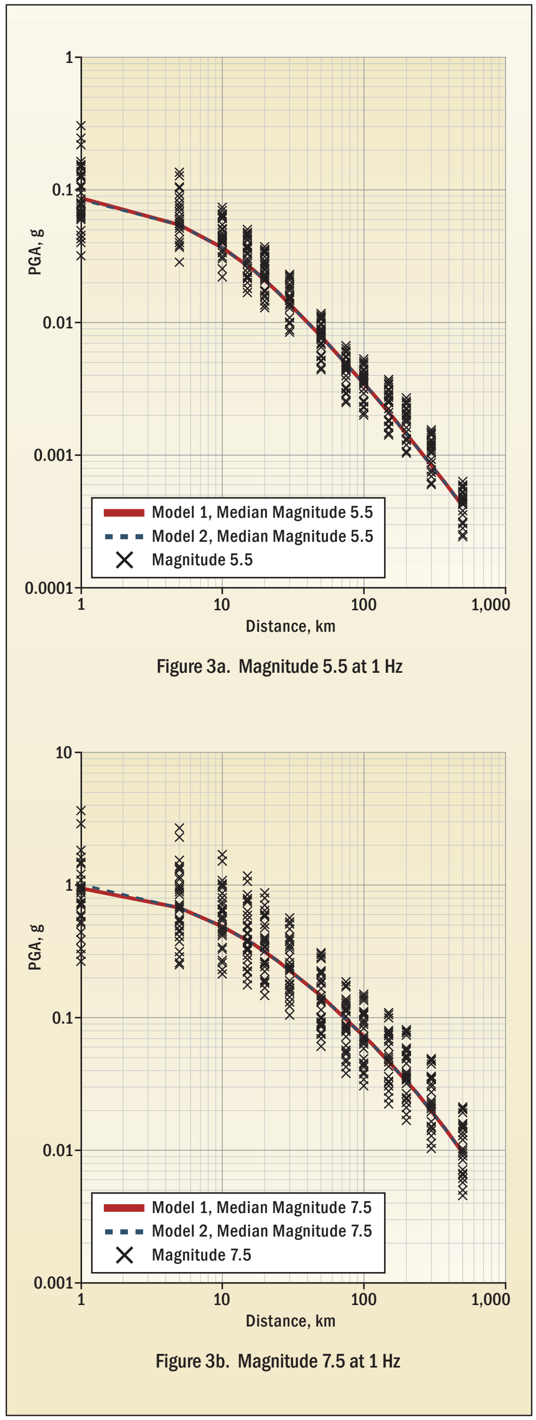

Figure 3a. Magnitude 5.5 at 1 Hz

Figure 3b. Magnitude 7.5 at 1 Hz

Figure 3. Comparisons Between Simulated Ground Motions and Two Regression Model Prediction Curves, as a Function of Epicentral Distance

The new toolbox allows for the efficient application of a region-specific attenuation model for seismic hazard studies.

This new streamlined toolbox allows the user to efficiently develop an applicable ground motion attenuation model for use in the seismic hazard assessment of a project, rather than being forced to make design calculations based on nonregion-specific models that may not be applicable. Typically, the required input parameters necessary to implement this methodology are available for regions of low seismicity even though empirically based ground motion attenuation models are not. Thus, the SMSIM MLREG toolbox may be used for any future project for which region-specific ground motion attenuation relationships for seismic hazard analysis are needed but are not otherwise available. ■

TRADEMARKS

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BIOGRAPHIES

Behrooz Tavakoli, PhD, has more than 15 years of experience in engineering seismology, geophysics, and engineering geology. He has worked at Bechtel for more than 3 years to develop and review site-specific ground motion design criteria for projects ranging from conventional structures under modern building codes to LNG and nuclear power plants under foreign and domestic regulatory provisions. His research interests focus on stochastic modeling of earthquake events, site hazard characterization, ground motion modeling, earthquake damage and loss estimation, risk analysis of structures, and synthetic earthquake records to simulate strong ground motions.

Behrooz has published several technical papers on topics related to earthquake seismology in peer-reviewed journals or conference proceedings. For example, he worked on the New Madrid Seismic Zone (NMSZ), located southwest of New Madrid, Missouri, to obtain the best estimation of earthquake occurrence rates and strong ground motions where causative faults of earthquakes are poorly understood. As a result, Behrooz has proposed a new 3D mechanical model of faulting to properly explain the occurrence of earthquakes in the NMSZ, to illustrate the potential rupture faults for the 1811–1812 earthquake sequences, and to open new areas of research. He also developed a new ground motion attenuation relationship for eastern North America. Both of these recent research results were published in the *Bulletin of the Seismological Society of America*. Behrooz has also been the principal investigator for the Global Seismic Hazard Analysis Program (GSHAP) that is producing a new earthquake hazard zonation map for the country of Iran.

Behrooz received his PhD in Seismology from Uppsala University, Sweden, and an MS in Geophysics and two BS degrees (Civil Engineering and Geology) from the University of Tehran, Iran. He was granted a 2-year post-doctoral research fellowship award in practical seismology from the Department of Civil Engineering at the University of Memphis, Tennessee, to pursue his research interests in the field of earthquake seismology and geotechnical earthquake engineering.

Nick Gregor, PhD, has been involved in the seismological analysis of various engineering sites throughout his 15-year career at Bechtel. He has broad experience in applying seismology to develop both probabilistic and deterministic ground motions (both seismic source modeling and ground motion attenuation models) and in applying building codes and regulations. Nick also has extensive experience in developing spectrum-compatible time histories for engineering design analysis. During his career, he has provided seismological consulting services for numerous projects worldwide, including hydroelectric dams, bridges, nuclear power plants, nuclear waste repositories, water and gas pipelines, rail lines, ports, landfills, hospitals, electric substations, and office buildings.

Nick has published several papers in peer-reviewed journals and has participated as a peer reviewer for manuscripts submitted for publication.

Nick received his PhD and AB, both in Geophysics, from the University of California at Berkeley. His doctoral thesis was on the development of a Peak Ground Motion Displacement attenuation model.