PROBABILISTIC SITE SEISMIC RESPONSE ANALYSIS

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Abstract—*Determination of seismic site response is the important first step in any earthquake-related engineering study. The most popular approach in current practice, adopted first in the SHAKE computer program in 1972, assumes that the ground profile consists of an assembly of horizontal soil/rock layers with different material properties. This approach requires selecting an acceleration time history as the representative motion for the design earthquake, usually through a spectral matching process from a recorded "seed" motion. Due to uncertainties at many steps of this procedure, the results of a site response analysis from a single time history record may scatter significantly. To get statistically stable results for a practical site such as a nuclear facility, it may be necessary to perform the same site response analysis many times using 30–60 different time histories. Thus, the procedure becomes cumbersome and very time consuming.*

An alternative procedure is proposed in this paper to overcome the shortcomings. In this new procedure, the original SHAKE framework of site response analysis is preserved. However, instead of using an acceleration time history as the seismic input, a design spectrum is used as the input motion directly. Power spectrum densities (PSDs) are calculated in each step of the procedure. Extreme values of stress, strain, acceleration, and response spectra are derived directly from the PSDs based on relationships obtained from random-vibration-theory (RVT). The results represent statistical means of the quantities of interest from all possible input time histories fitting the same design spectra. This alternative procedure is coded in a new program, P-SHAKE.

Numerical examples included in the paper demonstrate the compatibility of P-SHAKE results with the results of "conventional" SHAKE runs, and the efficiency and easiness of this new procedure in generating statistically meaningful and stable results. This new approach has been used successfully in the site response analysis work for several large-scale Bechtel projects.

Keywords—*P-SHAKE, random vibration theory (RVT), site seismic response analysis*

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INTRODUCTION

In current engineering practice, seismic design motions at the sites of most critical structures are developed by first generating the rock motions using probabilistic seismic hazard analysis (PSHA) and then by analyzing site response through the soil column to incorporate local soil effects. The most popular approach used in site response analysis, adopted first in SHAKE [1, 2] and its linear and nonlinear variations, assumes that the ground profile consists of an assembly of horizontal soil/rock layers with different material properties. One acceleration time history, which usually starts with a recorded motion as the "seed" and is modified to fit a given design response spectrum, is specified at a certain elevation, and the responses of the soil profiles, including stresses, strains, maximum accelerations, response spectra, etc., are computed. Soil nonlinearity can be considered through either equivalent linear or nonlinear numerical iterative procedures. **Figure 1** shows the approach schematically.

Uncertainties arise in using the above procedure on practical engineering projects. One major uncertainty is the appropriate selection of the input time history. It is well known that two acceleration time histories may fit the same design response spectrum but differ in other important characteristics such as velocity, displacement, Arias intensity, and power spectrum density (PSD). This is mainly due to the fact that the phasing and energy characteristics of the time history play significant roles in soil column responses, particularly for site conditions where

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ABBREVIATIONS, ACRONYMS, AND TERMS

P-SHAKE	Bechtel's alternative approach for site seismic response analysis based on RVT and working within the theoretical framework of SHAKE
PSD	power spectrum density
PSHA	probabilistic seismic hazard analysis
RVT	random vibration theory
SHAKE	a computer program for conducting equivalent linear seismic response analyses of horizontally layered soil deposits
UHS	uniform hazard spectrum

The current SHAKE approach can be very cumbersome and time consuming, especially for site response studies of critical structures, such as a nuclear plant.

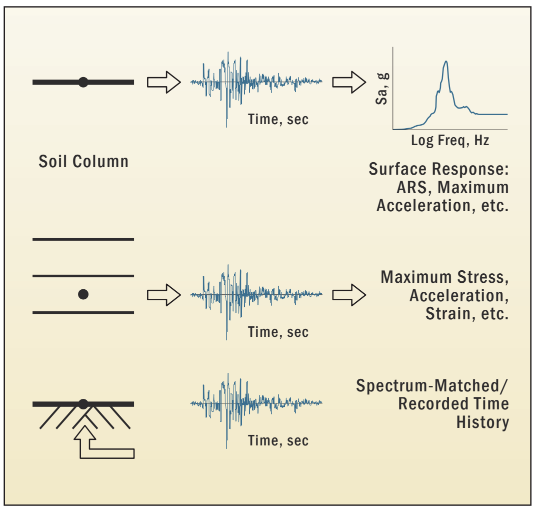


Figure 1. Site Response Analysis—Time History Approach

soil nonlinearity becomes important (e.g., see [3]). The most commonly used approach to reducing this uncertainty is to (1) generate multiple time histories, all fitted to the same design spectrum and orthogonal to each other but originated from different earthquake recordings; (2) run the site response computation many times, each time using a different time history; and (3) take statistic measures and bonding values from the multiple runs for the quantities of interest for the engineering project. This approach can be very cumbersome and time consuming, especially for site response studies of critical structures, e.g., a nuclear power plant, in which a group of 30–60 time histories is usually required to obtain statistically stable results. Selecting such a large suite of time histories at sites where only limited recorded motions are available (e.g., eastern US) is very challenging and often involves modifying the motions from other regions to fit the project site.

This paper presents P-SHAKE, Bechtel's alternative approach for conducting seismic site response analysis that eliminates the need to generate time histories. This approach follows the SHAKE theoretical framework but uses formulations based on random vibration theory (RVT) for input motion and soil column analysis. This new approach follows three basic steps:

- First, the input target rock response spectrum is converted to a PSD function.
- Next, the PSDs of the site soil column responses are computed based on the input PSD and the transfer functions of the soil column. The statistical means of the maximum shear strains and effective strains are obtained based on the PSD, and the process is repeated until strain compatibility is reached over the entire soil column.
- Finally, the PSDs and the statistical means of the maximum responses of other required quantities, such as the acceleration response spectra and maximum accelerations, are computed once convergence has been reached on soil properties.

Figure 2 shows the new approach schematically.

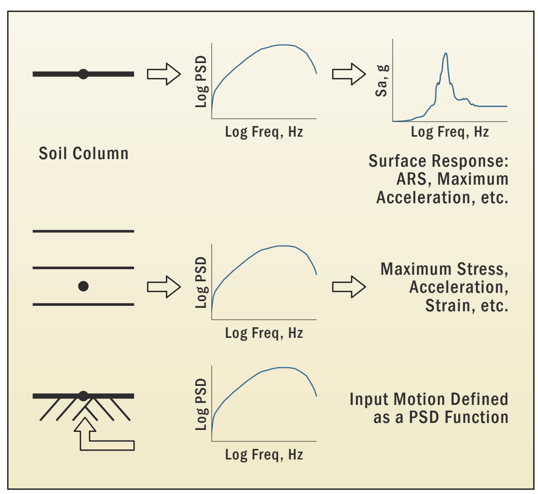


Figure 2. Site Response Analysis—Alternative Approach

Converting Acceleration Response Spectra to Power Spectrum Density Functions

It is well known from basic RVT theory (e.g., see [4]) that

$$S_d(\omega) = \left|H^2(\omega)\right| S_a(\omega), \quad (1)$$

where $S_d(\omega)$ is the relative displacement PSD, $S_a(\omega)$ is the acceleration PSD, $H(\omega)$ is the transfer function between displacement response and absolute acceleration input of a single-degree-of-freedom oscillator with frequency ω_o and damping ξ, and

$$\left|H^2(\omega)\right| = \frac{1}{(\omega_o^2 - \omega^2)^2 + 4\xi^2\omega_o^2\omega^2}. \quad (2)$$

The mean of the maximum relative displacement response of the oscillator (definition of a mean relative displacement response spectrum) is given by

$$D = p\sqrt{\lambda_0}, \quad (3)$$

where p is a peak factor and λ_0 is the zero moment of the response defined in Equation 6. Following References [5] and [6],

$$p = \sqrt{21n\upsilon(0)\tau} + \frac{0.5772}{\sqrt{21n\upsilon(0)\tau}}, \quad (4)$$

where $\upsilon(0)$ is the mean zero crossing of the response between 0 and τ (the strong motion duration of the earthquake), with

$$\upsilon(0) = \frac{1}{\pi}\sqrt{\frac{\lambda_2}{\lambda_0}}. \quad (5)$$

The moments of the response are defined as

$$\lambda_n = \int_0^\infty \omega^n S_d(\omega)d\omega, \quad (6)$$

where $n = 0, 1, 2$ for the zero (λ_0), first (λ_1), and second (λ_2) moments of the response, respectively.

Following References [7] and [8], $\upsilon(0)$ is necessarily adjusted with the parameter δ, where

$$\delta = \sqrt{1 - \frac{\lambda_1^2}{\lambda_0\lambda_2}}. \quad (7)$$

In summary, the acceleration PSD function from a given acceleration response spectrum is calculated in the following steps:

1. Convert the acceleration response spectrum $RS_a(\omega)$ to a relative displacement response spectrum $RS_d(\omega)$.
2. Assume an initial acceleration PSD function $S_{a,0}(\omega)$. Usually a constant value of unity is assumed as the initial value over the frequency range.
3. With the assumed $S_{a,0}(\omega)$ and the relations given above, calculate the mean of the maximum relative displacement response for all of the frequencies defining the response spectrum. This will be a new relative displacement response spectrum $RS_{d,1}(\omega)$.
4. Calculate the ratio $R(\omega) = RS_d(\omega)/RS_{d,1}(\omega)$.
5. Correct the assumed acceleration PSD function $S_{a,0}(\omega)$ by $R^2(\omega)$ to calculate a new acceleration PSD function $S_{a,1}(\omega)$.
6. Iterate steps 3 to 5 until the desired accuracy is reached in the calculation of the displacement response spectrum.

This paper presents P-SHAKE as an alternative approach for conducting seismic site responses that eliminates the need to generate time histories.

Determining the Mean of Maximum Responses

Using the acceleration PSD $S_a(\omega)$ of the input motion and the transfer function between the input and any desired response $H_r(\omega)$, which is calculated following the normal SHAKE procedure, the steps to calculate the mean of the maximum response are as follows:

1. Calculate the PSD of the desired response:

$$SR(\omega) = \left|H_r^2(\omega)\right| S_a(\omega). \quad (8)$$

2. Calculate the moments λ_0, λ_1, and λ_2 of the response:

$$\lambda_n = \int_0^\infty \omega^n SR(\omega)d\omega. \quad (9)$$

3. Calculate the peak factor p with these moments as described in Step 1.
4. Calculate the mean of the maximum response:

$$M_R = p\sqrt{\lambda_0}, \quad (10)$$

where p is the peak factor for the desired response, following the same procedure outlined in Equations 4 through 7 but with the response PSD from Equation 8.

NUMERICAL EXAMPLE

Bechtel has coded the above procedure in the computer program P-SHAKE. [9] An earlier study [10] demonstrated that P-SHAKE results generally are in very good agreement with SHAKE results for an individual earthquake time history. The following numerical example illustrates compatibility of the P-SHAKE results with the SHAKE analysis results and the efficiency of the new approach.

A 1,630-foot-deep soil profile consisting of various sand, clay, and soft rock layers overlaying a rock half-space was analyzed. The site shear wave velocity profile is shown in **Figure 3.** A group of strain-degradation curves for shear moduli and damping ratios is assigned to various soil layers but is not presented here due to space limitations.

The P-SHAKE results generally are in very good agreement with SHAKE results for an individual earthquake time history.

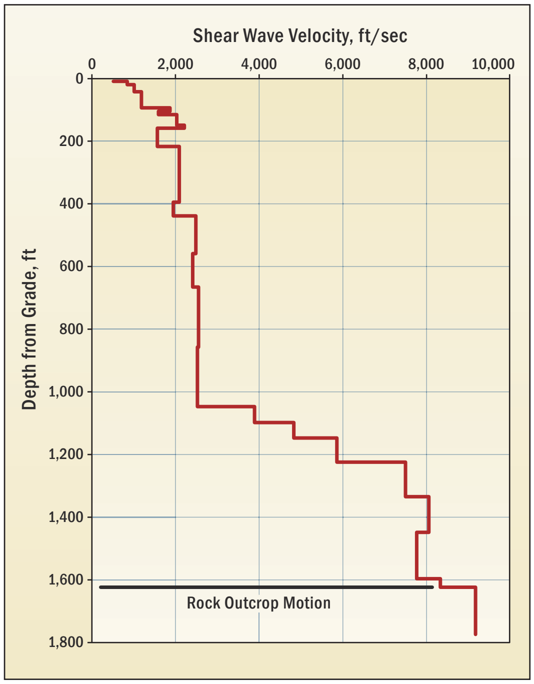


Figure 3. Shear Wave Velocity Profile of the Example Problem

The uniform hazard spectrum (UHS) at the rock surface outcrop with 10^{-5} recurrence period was developed through PSHA **(Figure 4).** Thirty acceleration time histories were selected as the "seeds" from historical recordings around the site and from other earthquakes with similar geological and seismological conditions. **Figure 5** shows the overall match of the 30 time histories with the rock UHS; **Figure 6** shows a few of these matched rock time histories.

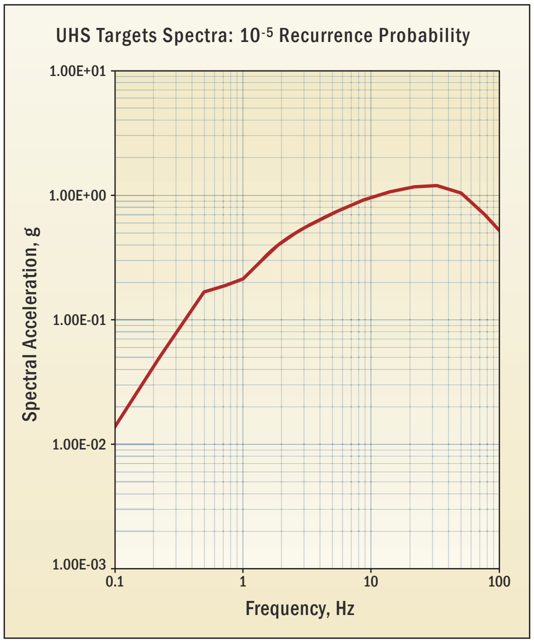


Figure 4. Uniform Hazard Spectrum of Rock Motion

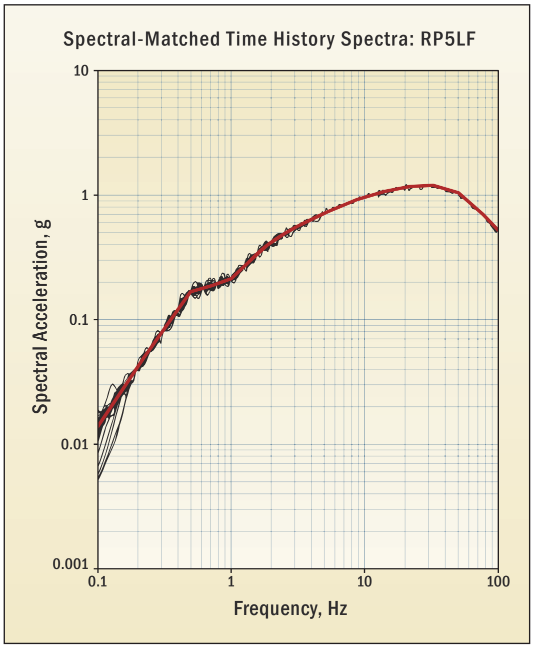


Figure 5. Target Spectra-Matching of 30 Time Histories

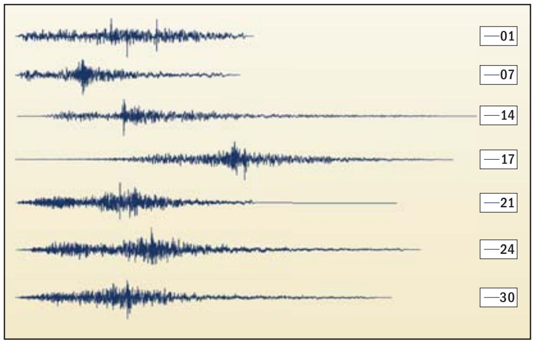


Figure 6. Selected Plots of Spectra-Matched Time Histories (Number in Box is the Sequential Number)

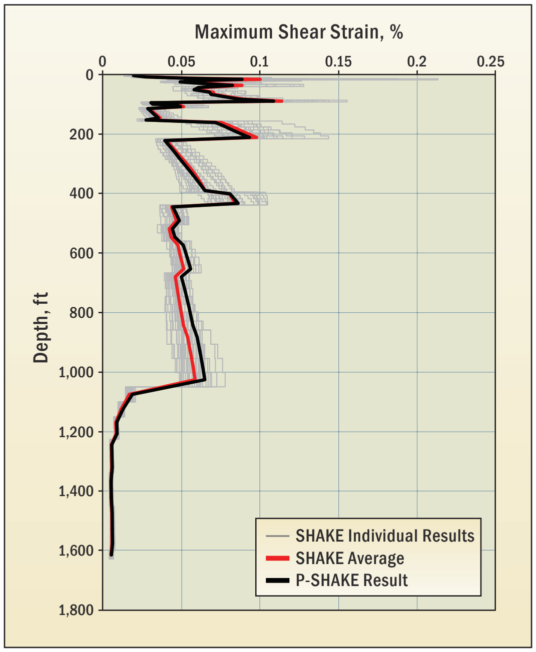


Figure 7. Computed Maximum Shear Strain Profile

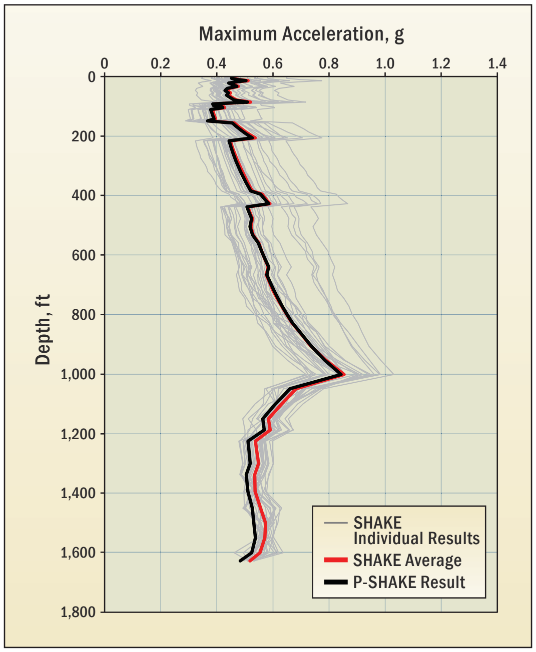


Figure 8. Computed Maximum Acceleration Profile

Two parallel analyses were performed. The first one used the SHAKE program, repeated 30 times using the matched time histories as input motions, one time history at a time. The second one used the P-SHAKE program, with the 5%-damped UHS as the input motion. In both cases, the input motion was specified as outcrop motion at the top of the rock half-space.

Figure 7 shows the maximum shear strains developed in the soil profile after convergence was reached on soil properties. **Figure 8** shows the maximum acceleration profile of the analysis results. **Figure 9** shows the acceleration response spectra at the ground surface. In all the figures, the thin gray lines are from the 30 individual SHAKE analyses, which show, as expected, large variations from the results of the different time histories. The thick red line is the average of all SHAKE analyses, and the thick black line is the P-SHAKE results. It can be observed quite

The SHAKE and P-SHAKE results are in very good to excellent agreement. However, SHAKE requires 30 time histories and 30 analyses for the same profile while P-SHAKE needs only one to achieve essentially the same results.

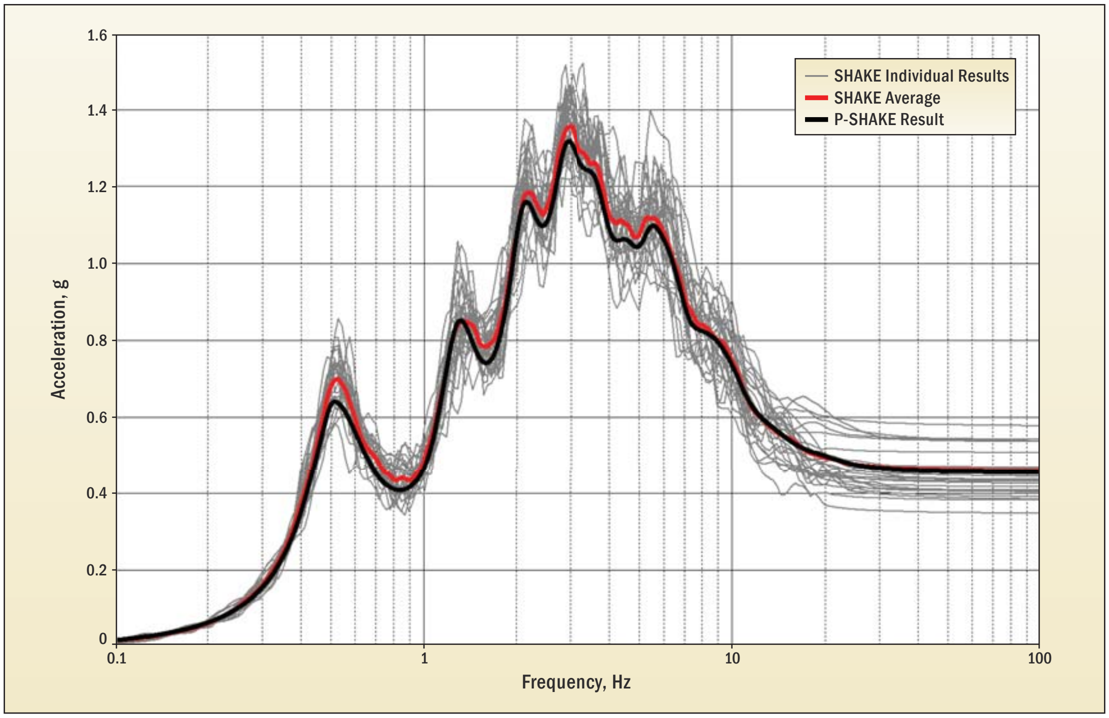


Figure 9. Acceleration Response Spectra at Ground Surface

P-SHAKE has been used successfully in the site response analysis for several major nuclear power plant sites and has been accepted by the US Nuclear Regulatory Commission.

clearly that the two sets of results are in very good to excellent agreement. However, SHAKE requires 30 analyses for the same profile while P-SHAKE needs only one to achieve essentially the same results.

Worth mentioning is that we have tested many cases in numerous different soil profiles with multiple time histories to demonstrate the compatibility and close agreement between SHAKE and P-SHAKE results. These results are not shown here due to space limitations. The P-SHAKE program is now widely used for major Bechtel projects.

CONCLUSIONS

An alternative approach for site seismic response analysis is presented in this paper. This approach is based on RVT and works within the theoretical framework of the computer program SHAKE. In this approach, the design input motion is characterized directly by the design response spectrum, all intermediate computations are calculated through PSD and transfer functions, and all responses of interest are calculated as statistical averages. The P-SHAKE approach avoids the difficulties associated with generating multiple spectrum matching input time histories and is most suitable to use with a suite of randomized soil profiles for soil amplification studies.

Numerical examples show that the results computed by the new approach are in good agreement in statistical average with the results computed by SHAKE. Thus, all practical engineering experiences and empirical relationships built upon SHAKE are still applicable. P-SHAKE has been used successfully in the site response analysis work for several major nuclear power plant sites and has been accepted by the US Nuclear Regulatory Commission. ■

REFERENCES

[1] P.B. Schnabel, J. Lysmer, and H.B. Seed, "SHAKE: A Computer Program for Earthquake Response Analysis of Horizontally Layered Sites," *Report No. UCB/EERC-72/12*, Earthquake Engineering Research Center, University of California, Berkeley, 1972, access via http://nisee.berkeley.edu/elibrary/Text/21000829.

[2] I.M. Idriss and J.I. Sun, *User's Manual for SHAKE91*, University of California, Davis, November 1992, refer to http://www.ce.memphis.edu/7137/PDFs/Notes/Shake91.pdf (August 1992) and/or http://nisee.berkeley.edu/elibrary/Text/360175 (1993).

[3] F. Ostadan, S. Mamoon, and I. Arango, "Effect of Input Motion Characteristics on Seismic Ground Responses," *Proceedings of the 11th World Conference on Earthquake Engineering (11WCEE)*, Acapulco, Mexico, June 23–28, 1996, http://www.iitk.ac.in/nicee/wcee/article/11_1924.PDF.

[4] A. Der Kiureghian, "Introduction to Random Processes," Lecture Notes for Short Course on Structural Reliability: Theory and Applications, University of California, Berkeley, March 23–25, 1983.

[5] A.G. Davenport, "Note on the Distribution of the Largest Value of a Random Function with Application to Gust Loading," *Institution of Civil Engineers (ICE) Proceedings*, Vol. 28, No. 2, June 1964, pp. 187–196, access via http://www.icevirtuallibrary.com/content/issue/iicep/28/2.

[6] A. Der Kiureghian, "Structural Response to Stationary Excitation," *Journal of the Engineering Mechanics Division*, ASCE, Vol. 106, No. 6, November/December 1980, pp. 1195–1213, access via http://cedb.asce.org/cgi/WWWdisplay.cgi?9906.

[7] T. Igusa and A. Der Kiureghian, "Dynamic Analysis of Multiply Tuned and Arbitrarily Supported Secondary Systems," *Report No. UCB/EERC-83/07*, Earthquake Engineering Research Center, University of California, Berkeley, 1983, access via http://nisee.berkeley.edu/elibrary/Text/131000583.

[8] E.H. Vanmarcke, "On the Distribution of the First-Passage Time for Normal Stationary Random Processes," *Journal of Applied Mechanics*, Vol. 42, No. 1, March 1975, pp. 215–220, access via http://asmedl.aip.org/dbt/dbt.jsp?KEY=JAMCAV&Volume=42&Issue=1.

[9] *User's Manual for P-SHAKE*, Version 2.0, Bechtel National Inc., September 2009.

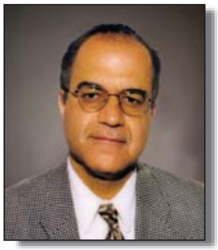
[10] N. Deng and F. Ostadan, "Random Vibration Theory Based Seismic Site Response Analysis," Paper No. 04-02-0024, *Proceedings of the 14th World Conference on Earthquake Engineering (14WCEE)*, Beijing, China, October 12–17, 2008, http://www.iitk.ac.in/nicee/wcee/article/14_04-02-0024.PDF.

BIOGRAPHIES

Nan Deng, PhD, has more than 20 years of experience in geotechnical and geotechnical earthquake engineering and foundation design. As a senior engineering specialist, he has actively participated in many of Bechtel's global civil, mining, and power projects and research activities, ranging from dams to nuclear sites to waste treatment plants. Nan specializes in all aspects of geotechnical analysis and design, especially seismic site characterization, liquefaction, and soil structure interactions.

Nan has published more than 15 technical papers on topics related to geotechnical earthquake engineering. He is currently a member of the American Society of Civil Engineers (ASCE), Geotechnical Division.

Nan received his MS and PhD in Civil Engineering from the University of California, Berkeley, and his BEng in Hydraulic Engineering from Tsinghua University of Beijing, China. He is a registered Professional Engineer in California.



Farhang Ostadan, PhD, a Bechtel Fellow, has more than 27 years of experience in geotechnical and geotechnical earthquake engineering and foundation design. As chief soils engineer for Bechtel, he has overall responsibility for this discipline and manages the efforts of a large and diverse group of geotechnical specialists in locations across the US and around the globe. His project oversight responsibilities range from major transportation projects to petrochemical, nuclear, and power- and energy-related projects.

Farhang has published more than 30 technical papers on topics related to geotechnical earthquake engineering. He co-developed a method for dynamic soil structure interaction analysis currently in use by the industry worldwide. Farhang is a frequent lecturer at universities and research organizations.

Farhang is a Fellow of the American Society of Civil Engineers (ASCE), Geotechnical Division; is a member of the Earthquake Engineering Research Institute (EERI); and was recently elected chairman of the international conference on Structural Mechanics in Reactor Technology (SMiRT 22) to be held in San Francisco in August 2013.

Farhang received a PhD in Civil Engineering from the University of California, Berkeley; an MS in Civil Engineering from the University of Michigan, Ann Arbor; and a BS in Civil Engineering from the University of Tehran, Iran.